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THE THERMOPHOSPHORESCENT RADIATIONS OF HIDDENITE AND KUNZITE

By Otto Stuhlman, Jr.

Hiddenite is a crystal mineralogically peculiar to North Carolina, a variety of spodumene, a Li Al $(Si\ O_3)_2$ containing[1,2] about 65% silicon, 27% aluminum, 7% lithium and 0.5% sodium. The remaining 0.5% made up of traces of iron, potassium and chromium. It is a small crystal resembling the emerald in brilliancy and color, the tint ranging from yellowish green to deep yellow emerald-green. It has a 6.5 hardness and a specific gravity[3] of 3.15 to 3.19.

Large crystals are very difficult to obtain and the small samples examined were slightly inclined prisms, ranging from quite small fragments to pieces less than 1 cm in length and about 5 mm in diameter. Crystals over 5 cm in length and 1 cm in diameter are very rare.

The interest centered around its thermophosphorescent radiations as compared with those from kunzite. The latter crystal is a lilac tinted variety of spodumene with physical and chemical composition similar to that of hiddenite but containing different traces of impurities to which its natural colors are attributed. The color of kunzite is thought to be due[4] to its 0.11% mangenese content. In addition to mangenese Davis' analysis[5] shows it to contain 0.06% potassium, 0.06% nickel and 0.44% zinc.

The chemical analysis of hiddenite as made by Genth[2] shows it to contain iron to the extent of 1.11%, of chromium 0.18% and of potas-

[1] Smith, J. L., Amer. J. Sc., *21*, p. 128; 1881.
[2] Genth, Amer. J. Sc., *23*, p. 68; 1882.
[3] N. C. Geo. and Eco. Survey, *12*, p. 37; 1907.
[4] Baskerville, Sc., *18*, p. 303; 1903, Sc., *18*, p. 769; 1903.
[5] Davis, Amer. J. Sc., *8*, p. 29; 1904.

sium 0.07%, while an earlier analysis by Smith,[1] shows it to contain no traces of either potassium or chromium.

This difference in analysis is important in view of the fact that Genth attributes the green color of hiddenite to its slight chromium content.

Since their thermophosphorescent spectra depend upon the presence or absence of traces of the above elements, it was found necessary to subject hiddenite to a spectroscopic examination before proceeding with an analysis of these radiations.

The crystal was first ignited in an oxy-gas flame. Its visible spectrum consisted of the very intense sodium D and red lithium lines, but no trace of a spectrum indicating the presence of potassium was noticed. A series of spectrograms, made on panchromatic plates, showed no faintest traces of potassium. The presence of the characteristic "raies ultimes" of potassium could not be established although a large group of fragments of hiddenite taken from six different samples of crystals and selected for their varying shades of yellow-green were examined.

The arc spectrum of the crystal was next investigated for potassium lines. The carbon rods used for the arc contained slight traces of iron. The spectrum produced by the crystal showed pronounced groups of lines attributable to lithium, sodium, iron and chromium. The presence of lithium was confirmed by four pronounced lines, that of iron by twelve and chromium by five, after all other questionable lines had been accounted for as attributable to the original carbon arc. No spectral lines associated with the arc spectrum of potassium could be identified. For this purpose a comparison spectrum of potassium was used. If potassium were present in any form, then such persistent lines as the "raies ultimes," would indicate at least the presence of potassium to the extent of one part in one hundred thousand, so that the .07% potassium found by Genth could not have been overlooked in the samples used in these experiments. The above, however, confirms the analysis of hiddenite as made by Genth in regard to the chromium content which Smith did not find in his samples. Hence the yellow-green colors of hiddenite can be attributed to the slight chromium and iron content, since the colors of the simple iron and chromium compounds are predominantly yellow and green.

Hiddenite does not lose its color until its temperature of ignition has been attained. This was found to be near 1900° C. This heat-resisting property of hiddenite aside from its color, hardness and scarcity, ought to place it in the catagory of the gem stones.

THERMOPHOSPHORESCENCE OF HIDDENITE

Near 1050° C hiddenite becomes luminescent, emitting a pale reddish glow which becomes more intense with rise in temperature up to the point of ignition. Visual examination with a Gaertner wave-length spectrometer established the presence of a band of low luminosity extending from just beyond .683μ to .589μ with a maximum near .650μ.

A series of spectrograms on panchromatic plates obtained by means of the above instrument, led to the identification of two bands. When the crystal was kept just below its point of ignition the thermophosphorescent emissions consisted of an orange band extending from .700μ to .589μ with a maximum at .647μ and a faint green band, merging into the orange band with its maximum near .540μ and its short wavelength end near .531μ. The limiting values were very difficult to establish even after an exposure of two and one-half hours.

Since the scarcity of the crystals makes a more extended investigation prohibitive in cost we had to content ourselves with the above data as collected from three successful spectrograms obtained from a 20, a 60 and a 120 minute exposure. These spectrograms were made when the crystal was heated at the outer edge of an oxy-gas blow pipe placed so as to graze the crystal on the side away from the slit. In order to guard against spectral contamination by the flame, comparison exposures were made lasting 1.5 times the exposure used in obtaining the spectrograms from the crystal's radiations. These showed that the flame contributed nothing to the spectrograms for exposures less than 120 minutes. The spectrograms obtained from the flame at 120 minutes exposure showed a slight blackening at the blue end of the spectrum. A series of spectrograms made by means of a quartz spectrograph showed no evidence of ultra-violet emissions attributable to the thermoluminesce of the crystal despite a two hour exposure for which "Seed 23" plates were used.

THERMOPHOSPHORESCENCE OF KUNZITE

In a pervious[7] communication the thermophosphorescence of kunzite was shown to begin at 168° C. Pochettino's[6] work showed that his samples of kunzite began their emission at 235° C. This discrepancy seemed too great to be attributable to the possible difference in origin of the crystals. Additional samples from Pala, California were procured

[6] Pochettino, Il Nuovo Cimento., *18*, p. 245; 1909; *1*, p. 21; 1911.
[7] Stuhlman, Jr., and Daniel, J.O.S.A. & R.S.I., *17*, p. 289; 1928.

through Tiffany and Co., of New York. These showed a detectable glow which in the opinion of several observers began at about 210° C. If the crystals were held at the distance of distinct vision and viewed without a condensing lens the first detectable glow occurred near 230° C This latter observation is in agreement with Pochettino's results, the implication being that his observations were made when the crystal was held 25 cm from the eye.

A more detailed study of the change in color of the emissions resulted in the following color, temperature classification. If the temperature range was from 210° to 218° C the color was predominantly pale green, from 265° to 279° a yellow tinge was added, at 303° C the yellow became more saturated and remained unchanged until the temperature rose to 322° C. At 349° to 356° C the maximum visual brightness occurred accompanied by a reddish tinge superimposed on the earlier yellow-green shade. The composite effect at 380° C was an orange hue with more green appearing at higher and higher temperatures with accompanying decrease in brilliancy until at temperatures ranging from 370° to 420° C the crystal ceased to be luminescent.

Spectrograms of the radiations were not obtainable, although a large number of crystals were examined. An exposure of 18 hours, made by exhausting 18 pieces of crystal, each exposed for one hour, at a temperature ranging from 375° to 380° C produced no photographable results. A 15 hour exposure made at 395° to 402° C and a total photographic exposure of 30 hours from 240° to 380° C likewise produced no results. The cumulative intensity of the emission even after such an exposure was obviously below the threshold value of the panchromatic plate. An exposure made with an ultra-violet spectrograph using "Seed 23" plates exposed for six hours, likewise failed to show any emission bands.

When the light was examined through a carefully graded series of color filters, it was found that near 300° C the crystal emitted a yellow-green band extending from .515μ to .528μ with a maximum near .520μ and an orange band extending from .570μ to .650μ with a maximum near .593μ. At 400° C the green band was slightly wider with its maximum near .522μ. This slight change in the maximum could not be established in every case. The orange band, however, widened perceptibly; extending from .578μ to .680μ with a shift in the maximum to .605μ. Since the eye is much more sensitive to shades of green[9] near .510μ than to those of red, it follows that near 400° C the crystal apparently developed a greener shade due to the shifting of the maximum of the orange band to the longer wave-length region.

CHARACTERISTICS

The above characteristics have their counterpart in the emissions from the various oxides studied by Nichols and Howes[8] in that the crystals possess a definite temperature of maximum intensity. The radiations are subject to fatigue and their spectral distribution is dependent on the chemical character of the traces of impurities present. The emissions are also modified by previous heat treatment of the crystal.

The writer is indebted to the Rockefeller Grant for Scientific Research for partly financing this work and to Mr. Milton Braun for his able assistance in making the spectrograms.

THE UNIVERSITY OF NORTH CAROLINA,
CHAPEL HILL, N. C.

[8] Selected Topics in the Field of Luminescence, Merritt, Nichols and Child., National Research Council Bull., *5*, 1923.

[9] Hecht, and Williams, J. G., Physiology Rockefeller Inst. Med. Res., *1*, 1922–23.

THE PHOTO-EMF IN SELENIUM*

By Robert L. Hanson

I. INTRODUCTION

1. The presence of a photo electromotive force within a selenium cell was first observed in 1876 by Adams and Day[1] who found that in certain cases a cell upon being illuminated by limelight exhibited a current without the aid of an external battery. Uljanin,[2] using a cell made by placing a very thin sheet of selenium between transparent platinum films, obtained an electromotive force as high as 0.12 volts when the cell was exposed to sunlight. Righi,[3] working with a cell made by placing a wire gauze over a sheet of selenium melted on a brass plate, found that after a proper heat treatment the cell showed an emf between 0.085 and 0.115 volts when it was exposed to the illumination of a gas lamp. He found that sunlight gave about one and two thirds times as great an effect.

2. Coblentz[4] has made a detailed study of the electromotive force caused by short wave radiation on molybdenite crystals in which he finds a rather wide variation of the emf depending upon the part of the crystal receiving the radiation.

II. PURPOSE OF THIS WORK

1. The experiments here described were carried out in order to obtain more definite knowledge regarding this photo-emf in polycrystalline selenium—an effect which was first brought to the attention of the writer while making a study of the variation of the thermo-electric power of selenium when exposed to the bombardment of electrons. In this work the cells were used in a vacuum system which contained a mercury diffusion pump and a mercury trap. Even though the diffusion pump was not being operated, a cell having a resistance as high as 100 megohms, when left in the vacuum system without liquid air trap, would become sufficiently contaminated with mercury in a period of two or three hours to lower its resistance to a few thousand ohms. An investigation of the change in conductivity of this cell upon being il-

* A thesis presented to the faculty of the graduate school of Cornell University for the degree of Doctor of Philosophy.

[1] Proceedings of the Royal Society of London, *25*. p. 113; 1876.

[2] Annalen der Physik, *34*, p. 241; 1888.

[3] Beiblatter zu den Annalen der Physik, *12*, p. 683; 1888.

[4] Scientific Papers of the Bureau of Standards, *19*, p. 375; 1923.

luminated revealed a light negative effect rather than a light positive effect as is usually the case, that is, its resistance increased with illumination. This phenomenon has previously been reported by Brown.[5] A more careful examination, however, showed that the "negative" effect in this cell was probably due not to an increase in resistance, but to an emf developed within the cell, for if connected directly across a galvanometer and illuminated, a deflection of several divisions was obtained in such a direction as to account for the apparent increase in resistance. As a matter of fact, it was later found that the actual resistance of this cell no longer showed any change with illumination, the apparent change being due to a photo-emf only.

2. Further experiments on cells made with pure selenium showed that in the majority of cases, the cells showed a photo-emf which seemed to be roughly of the same order of magnitude regardless of the heat treatment to which they had been subjected. The maximum emf obtained by illumination with a 400 watt lamp fell far below that reported by previous writers, being of the order of 0.03 volts. The use of filters necessary to take out the infrared radiation cut the available intensity down so that the effects studied were never higher than .005 to .010 volts.

3. Since this effect may have an important influence on the measurement of the thermal emf it seemed advisable to subject it to a careful study with the following points in view:

(a) Dependence of the photo-emf on the current through the cell.

(b) Effect of the intensity of the incident light.

(c) Effect of the wave length of the incident light.

(d) Relation between the photo-emf and photoelectrical conductivity.

III. APPARATUS

1. Three types of cells were used in this work.

(a) The first, type A, consisted of two strips of nickel cemented on a strip of ground glass and selenium smeared into the space between the nickel strips to a depth of about .5 mm, thus forming a cell with a current path about 2 mm long and 4 cm wide (see Fig. 1, A.) The cell was then placed in a brass tube, immersed in an annealing bath, slowly brought to a temperature ranging from 130° to 200°C, maintained at this temperature for a period of one or two hours and then cooled

[5] Physical Review, *2*, p. 153; 1913.

slowly. This annealing process converted the black amorphous, non-conducting material into the grey conducting crystalline form. The annealing temperature together with the time of annealing determines the resistance and the light sensitivity of the cell with respect to its change in electrical conductivity. A cell of this type annealed as described at a temperature of 130° for one hour will have a dark resistance of the order of one hundred to one hundred and fifty megohms and is very light sensitive, its conductivity changing by a factor of from 5 to 10 upon exposure to light. The same cell annealed at a temperature of 180° or 200°C for one hour may have a dark resistance as low as one megohm but in general will show a very low light sensitivity. Usually

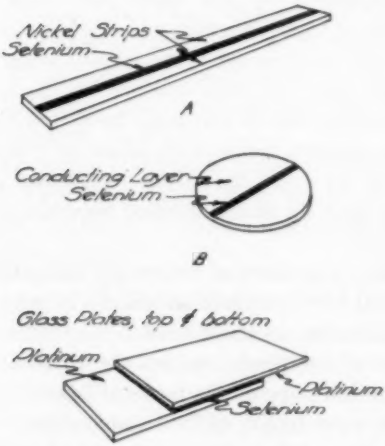

FIG. 1. *Cells used for measurement of photo-emf.*

the higher the annealing temperature and the longer the time of annealing, the lower will be the resistance and light sensitivity.

(b) The second form of cell studied, type B, was a commerical form, manufactured by Adam Hilger, Ltd. for spectroscopic use. It consists of a strip of polycrystalline selenium about one millimeter long and two centimeters wide. Contacts are made by a conducting layer the nature of which is not known to the writer (see Fig. 1, B.) This cell has a dark resistance of about 10,000 ohms.

(c) The third, type C, was similar in construction to the cells described by Uljanin.[2] Two pieces of plain glass were heated to the melting point of selenium and a small quantity melted on to one. The second

glass plate was placed on the molten selenium and the two pressed together forming a thin, uniform sheet with plain surfaces. Clamps we e placed on the plates and the whole annealed as previously described. The result was a thin, uniform sheet of selenium which with care could be cut up into several smaller pieces, thereby permitting the use of everal cells containing specimens which had been subjected to the same heat treatment. These thin sheets were then placed be-

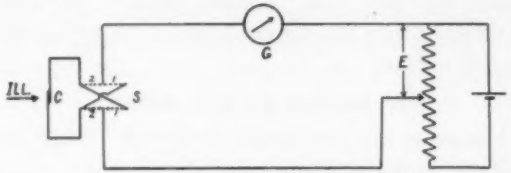

FIG. 2. *Electrical circuit. C, cell; G, galvanometer; E, applied potential; S. Reversing switch.*

tween two semi-transparent platinum films sputtered on glass, forming a cell with a conducting path ranging from .1 to .2 mm long and about 4 square centimeters in cross section (see Fig. 1, C.)

2. Fig. 2 shows the scheme of the electrical circuit used. An electromotive force E taken from a Wolff potentiometer was applied across a selenium cell C, the current being measured by means of the galvano-

FIG. 3. *L, 400 watt lamp; S. screens; W, water cell; F, filter; B, light tight box; D, movable carriage carrying thermopyle T and cell C.*

meter G which had a sensitivity of around 5×10^{-10} amperes per centimeter.

3. Throughout this work the intensity of illumination was measured by means of a Coblentz thermopyle and a D'Arsonval galvanometer, the intensity being varied by changing the distance between the selenium cell and the source of illumination.

4. Fig. 3 gives the general arrangement of apparatus. *D* is a movable carriage which carries the selenium cell *C* and the thermopyle *T* in

such a way that the cell or the thermopyle can be moved into the path of the light at will, thereby permittng the measurement of the energy exactly as it struck the cell. L represents a 400 watt tungsten projection lamp which was used as a source of illumination. After passing through diaphrams S the light passed through a water cell W before striking the cell or thermopyle. The carriage carrying the cell and thermopyle was enclosed in a light-tight box, the only opening beng E, the window through which the light from the lamp passed. Suitable light filters could be interposed into the beam of light at F for obtaining light in narrow spectral regions.

IV. PROCEDURE AND RESULTS

1. Let e represent the photo-emf in the cell. Then by use of the above circuit we get the relation

$$E+e=iR$$

provided the photo-emf is in the same direction as the impressed emf E, i being the current as measured by the galvanometer and R the total resistance of the circuit. The resistances of the cells used in this work were large so that the resistance of the galvanometer and the rest of

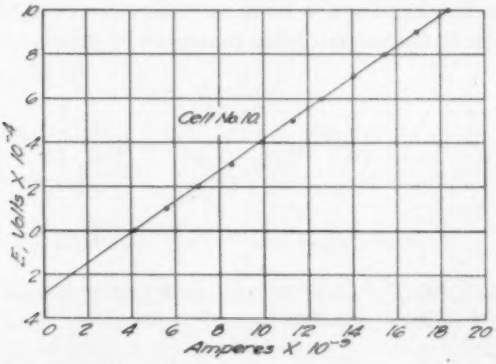

FIG. 4. *Curve showing applied emf plotted against the current through the cell.*

the circuit could be neglected and R considered as the internal resistance of the cell alone. e could easily be measured by use of the potentiometer in the ordinary manner but in this work the value of R was desired for all conditions under which e was measured.

If e is independent of the impressed emf E, or the current i through the cell, then varying E and plotting it as a function of i, a straight

line should result unless both R and e vary in such a manner that they just compensate each other. This seems highly improbable and it will later be shown that it is not true. Fig. 4 shows the results of varying the applied emf on cell No. 10 from zero to one millivolt while the illumination was kept constant. Similar runs were taken for several cells of the types described above for applied emf's ranging from zero to ten millivolts and for various intensities of illumination; out of more than a hundred such runs, never did there seem to be any tendency to deviate from a straight line. It is evident, therefore, that at least for the range above mentioned the photo-emf does not depend upon the applied emf or the current through the cell.

2. Two methods were used for measuring the photo-emf and resistance of the cells. The first was the following: a certain emf E such that $E>e$ was applied to the circuit shown in Fig. 2 with the switch S in position 1. In this case, let E and e be in the same direction. The current i was noted by means of the galvanometer. S is then put in position 2, thus making E and e opposing and the new current i' noted. From the two equations obtained for these two cases, namely,

$$E+e=iR$$
$$E-e=i'R$$

we get $e=(i-i'/i+i')E$ and $R=(2E/i+i')$. In this way R and e could be obtained from two galvanometer readings. This method has the disadvantage that such a determination of R and e is dependent upon two readings only and hence an experimental error in either will have a large influence on the results obtained. A more accurate method of obtaining R and e which was used for most of this work was that of varying E and plotting it as a function of the current i through the cell. The slope of the resulting line would give the actual resistance R, and e would be obtained from the intercept. The switch S may be in either position, tbe only difference being a positive or negative intercept for e.

3. For the determination of the photo-emf as a function of the intensity of illumination, the second of the above methods was used. For each intensity, a complete run was made varying the impressed emf E, plotting it as a function of current and using the intercept and slope for the determination of the photo-emf and the resistance respectively. Fig. 5 shows such a plot for cell No. 11 which was a commerical cell of type B previously described. Each curve is for a different

intensity of illumination. The first column of Table 1 gives the values of the photo-emf's obtained from this set of curves. The second and third columns give the intensities and the resulting resistances for the five curves here shown. The intensities are given in centimeters deflection of the thermopyle galvanometer. It is interesting to note that while the emf varies considerably with different intensities of illumination, the electrical resistance varies but little. The fact that the points on the current axis which represent the current through the cell when

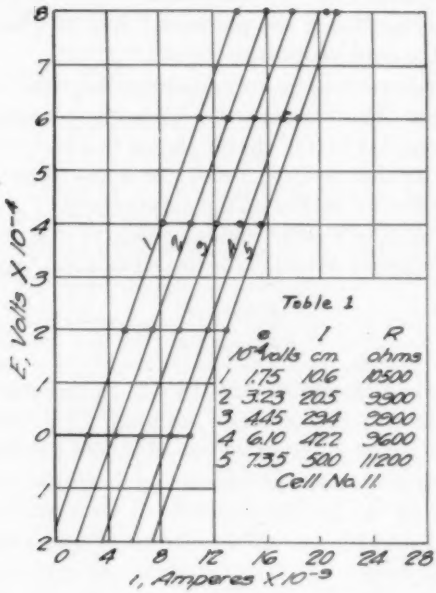

FIG. 5. *Applied emf plotted against current through the cell for different intensities of illumination.*

$E=0$ fall so consistently on the line of the other points substantiates the belief that the photo-emf is not affected by an external impressed emf.

4. Fig. 6 shows the photo-emf's obtained from the curves of Fig. 5 plotted as a function of intensity of illumination. This indicates that over a very wide range of intensities, the photo-emf bears a linear relation to the intensity of illumination. If the straight line be produced back to the axis of emf's, it will be noted that it indicates a

small emf for zero intensity. This, however, is contrary to the observed results. In no case has there been detected the slightest emf when the cell was not illuminated but the lowest intensities used give points

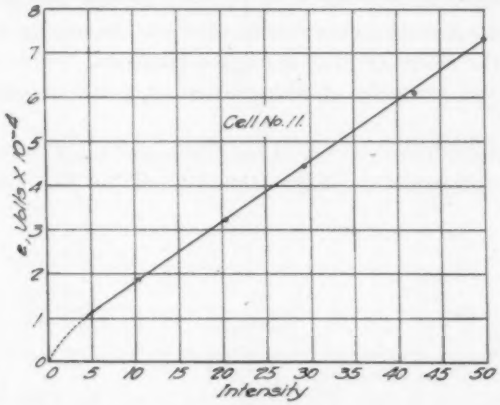

FIG. 6. *Photo-emf plotted as a function of the intensity of illumination incident upon the cell.*

lying directly on the straight line. The difficulty of obtaining accurate results for very weak intensities has thus far prevented a careful study

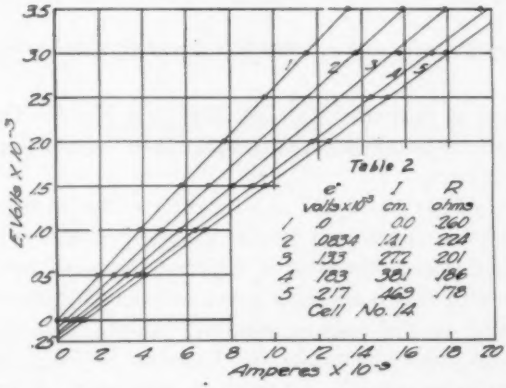

FIG. 7. *Applied emf against current through cell No. 14 for different intensities of illumination.*

of the effect near the origin. Whether the emf rises very suddenly in the manner of a trigger action or rises gradually to the straight line as indicated by the dotted curve is yet to be determined.

5. The curves in Fig. 7 show the impressed emf again plotted against the current for cell No. 14, a cell of type C previously described. Curve 1 is for no illumination and the lack of any intercept is illustrative of many determinations to detect if possible an emf in the dark. The decreasing slope of the curves indicates the decreasing resistance of this cell as the intensity of illumination increases. Table 2 shows the photo-emf, the intensity of illumination and the resistance for the five curves.

6. Fig. 8 shows the variation of the photo-emf and also the resistance for different intensities of illumination for cell No. 14. The emf curve

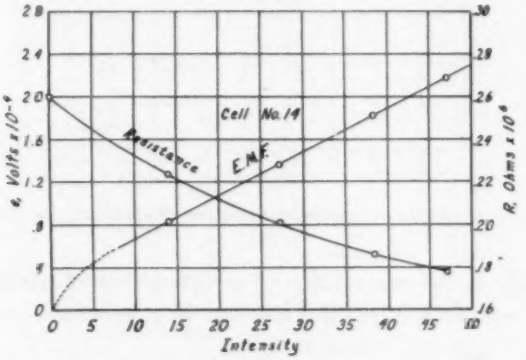

FIG. 8. *Curves showing the relation of photo-emf and resistance to the intensity of illumination for cell No. 14.*

is very similar to that shown for cell No. 11 in Fig. 6. The resistance curve is of the type which has generally been found for selenium by other investigators.

7. A study of the photo-emf as a function of the wave length of the incident light was carried out for the visible spectrum. Owing to the low intensities obtainable from a dispersed beam of light, filters in conjunction with the same lamp previously employed were used for obtaining light in the various spectral regions. For this purpose, Wratten filters manufactured by the Eastman Kodak Company were used in combination with solutions of potassium dichromate and nickel chloride. Table 3 shows the way in which the filters were used in combination with the two solutions mentioned to give the bands having a maximum percentage transmitted wave length noted in the second column of the table.

Keeping the source of light constant a run was taken for each filter or combination of filter and solution by varying the applied emf and plotting it as a function of current, the resulting straight line being used to determine the photo-emf and resistance as previously described. The carriage carrying the selenium cell and thermopyle was

TABLE 3

Filter	Wave length
70	700
71a + $NiCl_2$	630
72 + $NiCl_2$	600
73 + KCr_2O_4	570
74	530
75	490
76 + KCr_2O_4	440

then moved so that the thermopyle was in the path of the light and the energy of the transmitted beam through each filter observed. Assuming the previous determination of the linear relation between the photo-emf and intensity of illumination, the emf actually observed for each filter was divided by the energy transmitted, thus obtaining the emf on the basis of the same energy for each filter. The first and second

TABLE 4

Filter	Observed emf	Relative energy transmitted	Emf per unit energy
70	3.50×10^{-4} volts	1.00	3.50×10^{-4} volts
71a $NiCl_2$	1.25	.355	3.52
72 $NiCl_2$	0.81	.227	3.57
73 KCr_2O_4	1.80	.493	3.65
74	2.00	.524	3.83
75	2.25	.522	3.40
76 KCr_2O_4	1.00	.364	2.74

columns in Table 4 give the filters used and the actual emf observed respectively. The third column shows the relative transmitted energy expressed in arbitrary units, taking that for filter No. 70 as unity. The last column, obtained by dividing the values in the second column by those in the third, gives the emf per unit of energy.

The error introduced by making these calculations on the basis of a direct proportionality between the photo-emf and intensity is well within the limits of experimental error.

Curve A Fig. 9 shows the way in which the photo-emf varies with wave length. The exact form of the curve was not identical for all cells investigated; however, the general shape was the same. In all cases there was found to be a decided maximum in the region of $\lambda = 490 m\mu$. In many cases the emf for $\lambda = 440 m\mu$ was not as low compared to the value for $\lambda = 700 m\mu$ as was found to be the case here, but while there always appears to be a definite minimum at this end of the spectrum, it is often larger than the minimum at $\lambda = 700 m\mu$. Curve B shows the way in which the conductivity of the cell varied for different wave lengths. Curve C shows the general form of the absorption curve for polycrystalline selenium (taken from "Das Selen" by Ries). It is interesting to note that the maximum sensitivities for the photo-emf and the photoelectric conductivity occur at opposite ends of the visible spectrum. Comparing curves *A* and *C*, it appears that at least to a certain degree the photo-emf is developed by the wave lengths which have a high absorption.

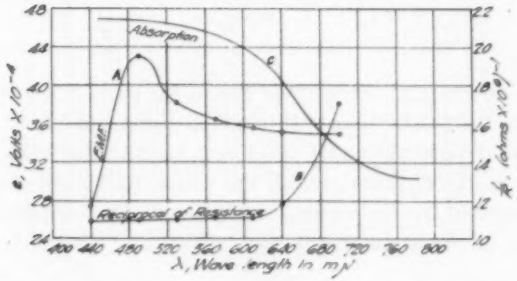

FIG. 9. *Curves showing the relation between the photo-emf, the reciprocal of the resistance, and the wave length of the incident illumination.*

3. The question at once presents itself as to whether the results of this effect could not be explained as a thermal-emf due to the heating effect of the light. This was answered by the following series of experiment.

(a) While in all cases the light had passed through a water cell before striking the selenium, several runs were made with a special heat filter manufactured by the Corning Glass Works introduced into the path of the light with no difference in the results.

(b) A piece of brass was heated to a red heat and placed in the position of the source of illumination without the evidence of any emf in the cell although the thermopyle gave a large galvanometer deflection.

(c) With the circuit open the cell was exposed to the illumination for ten to fifteen minutes thereby giving plenty of time for any heating of the exposed face of the cell; the switch was then closed and the time for the galvanometer to reach its final deflection noted by means of a stop watch. With the circuit closed the cell was shielded from the illumination and allowed to remain until thermal equilibrium could be established, after which it was exposed to the illumination and the time for the galvanometer to reach its full deflection again taken. In several trials there were no perceptable difference in the two observed times.

4. The photo-emf sensitivity differs from that of the photoelectrical conductivity in that it appears to be an instantaneous effect, as is shown from the two following experiments:

(a) The illumination was allowed to strike the cell and after the galvanometer had reached its maximum deflection it was found that if the hand be quickly passed through the beam of light there was a very decided kick in the galvanometer. This kick was much quicker and more pronounced than is found due to a change in the resistance by a sudden interruption of the illumination.

(b) The switch S, Fig. 2, was opened and a very intense light allowed to fall on the cell for several minutes. The light was then turned off and the switch immediately closed. If there were any lag in the photo-emf response one would expect to get a deflection of the galvanometer. This experiment was tried several times for each cell studied and in no case was there any evidence of a photo-emf.

5. These results of the study of the photo-emf in selenium agree very well in several respects to those obtained by Coblentz for the "actinoelectric" effect in molybdenite. They may be briefly summarized as follows:

(a) In both selenium and molybdenite it was found that an emf due to illumination was present in many specimens which showed little or no change in electrical conductivity.

(b) The variation of the photo-emf sensitivity with the wave length of the incident light which was found for selenium in the visible spectrum is very similar to that for several molybdenite crystals studied by Coblentz over the same region.

(c) For both materials the photo-emf response appeared to be practically instantaneous while there often appears to be considerable lag in the change in conductivity caused by illumination.

VI. SUMMARY

1. The photo-emf in a selenium cell over a wide range is independent of the current through the cell and bears a linear relation to the intensity of illumination.

2. The photo-emf sensitivity has a maximum for the visible spectrum in the region $\lambda = 490 m\mu$.

3. It is conclusively shown that the photo-emf can not be a thermal-emf.

4. No direct relation between the photo-emf and the photoelectrical conductivity effect is in evidence.

5. Comparisons are made between the results of this investigation and those obtained by Coblentz on the study of "actinoelectric" effect in molybdenite.

The author wishes to express his appreciation to Professor F. K. Richtmyer for his interest and many helpful suggestions. This work was carried on at Cornell University.

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THERMOLUMINESCENCE EXCITED BY HIGH VOLTAGE CATHODE RAYS

By Frances G. Wick and Edna Carter

Cathode rays, x-rays and the radiations from radioactive materials are known to produce marked luminescence. During exposure to these radiations many natural crystals and certain synthetic substances absorb energy which they give out only slowly, if at all, at ordinary temperatures. When the temperature is raised, they glow the more brightly and emit the energy the more rapidly the higher the temperature. Lenard and others have shown that there are limiting temperatures for different phosphors and for different bands of the same phosphor below which emission ceases entirely.[1] These substances may be completely deactivated by infrared radiation or by sufficient heating, and it has also been shown that exposure to the near ultraviolet radiation quenches the thermoluminescence.[2] The thermoluminescence of natural crystals is probably due to previous exposure to the radiations from radioactive substances, for in certain cases these crystals have been found to contain traces of radioactive material. The luminescence produced by alpha, beta and gamma rays has been studied by Przibram[3] and others. Wiedemann and Schmidt[4] prepared certain "solid solutions" which may be given a very intense thermoluminescence by exposure to cathode rays. The thermoluminescence produced by x-rays in certain natural crystals and also in these Wiedemann thermophosphors has been the subject of previous investigations.[2,5,6,7]

Since the penetrating cathode rays obtained at high voltages by Dr. Coolidge produce in many materials a much more intense luminescence than that due to other sources of excitation, it was thought to be of interest to study the thermoluminescence produced by these rays in some of the same materials whose thermoluminescence after exposure to x-rays had already been studied. The generous cooperation of Dr. Coolidge and the assistance of Dr. C. N. Moore made it possible

[1] Pringsheim, Fluorescenz und Phosphorescenz, 1928.
[2] Wick and Slattery, J.O.S.A. & R.S.I., *16*, pp. 398–408; 1928.
[3] Przibram, Wiener Ber. (IIA), *134*, p. 233; 1925.
[4] Wiedemann und Schmidt, Ann der Physik und Chemie, *56*, pp. 201–254; 1895.
[5] Wick, J.O.S.A. & R.S.I., *14*, pp. 33–44; 1927.
[6] Wick and Slattery, J.O.S.A. & R.S.I., *14*, pp. 125–132; 1927.
[7] Nichols, Howes and Wilbur, Cathodo-luminescence and the Luminescence of Incandescent Solids, Publication No. 384, Carnegie Institution, 1928.

to expose substances to cathode rays produced at voltages ranging from 150,000 to 240,000 in the research laboratory of the General Electric Company.

The specimens selected for examination were: (1) Franklin fluorite obtained from Ward. In its natural condition, this material possesses a green thermoluminescence which is intensified and changed in character by exposure to x-rays or cathode rays. (2) Ingleside calcite kindly furnished by W. P. Headden of the Colorado Agricultural College. This has a natural orange thermoluminescence which is greatly intensified by exposure to x-rays or cathode rays. (3) Calcium sulphate plus manganese, a Wiedemann thermophosphor prepared by adding a small amount of manganese sulphate and sulphuric acid in such proportions as to make one molecule of manganese to 250 of the sulphate as previously described.[6]

The specimens were powdered in a mortar and sifted through gauze of 70 or 100 mesh. The material was spread on a copper holder in a layer one mm thick and exposed in a horizontal position to a vertical beam of cathode rays coming through the window of a Coolidge cathode ray tube. The distance from the window to the specimen was about 5 cm. Unless otherwise stated, the voltage on the tube was 240,000, the current through the tube 1 milliampere and the time of exposure 10 seconds. The specimens were placed in glass tubes immersed in liquid air immediately after exposure in Schenectady and carried in large Dewar flasks to Vassar College, where a study of the thermoluminescence was made immediately upon their removal from the liquid air. The specimens remained unchanged so long as they were kept at this low temperature.

Photometric observations of the maximum brightness and the rate of decay of the thermoluminescence were made by the method used in previous experiments,[5] and the effects observed of varying the length of exposure time, the voltage on the cathode ray tube, the temperature of the substance during exposure and the temperature of observation.

Observations of maximum brightness at different temperatures of observation with varying lengths of exposure time indicate that, in general, saturation is obtained after a very short exposure, 10 seconds being sufficient in most cases.

EFFECT OF VARYING THE VOLTAGE ON THE CATHODE RAY TUBE

Voltages of 150,000, 200,000 and 240,000 were applied to the tube keeping the current at 1 milliampere. The specimens exposed at

100,000 volts were practically unaffected. Either the electrons did not get through the window or they had too little energy left to excite luminescence. The maximum brightness increases with increase in voltage, the change in almost every case being greater between 200,000

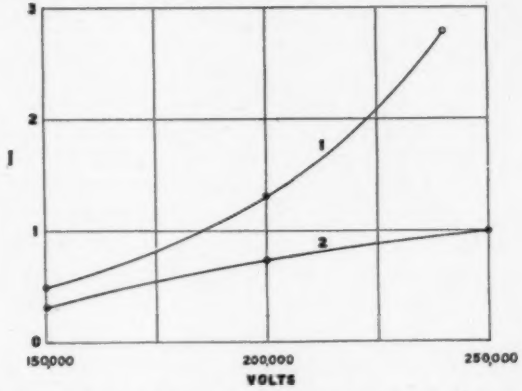

FIG. 1. *Effect of voltage of exposure upon maximum brightness of calcium sulphate plus manganese:* (1) *Temperature of observation 165°C*; (2) *Temperature of observation 110°C.*

and 240,000 volts than between 150,000 and 200,000 volts, especially at the higher temperatures of observation. The effect of variation in voltage upon maximum brightness of calcium sulphate plus manganese

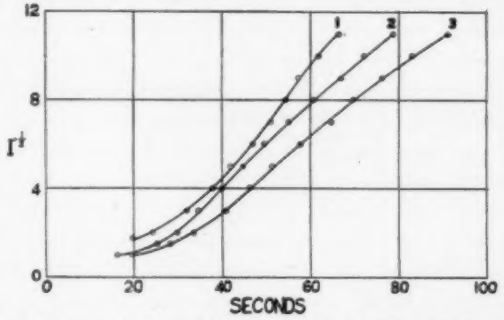

FIG. 2. *Effect of voltage of exposure upon decay curves of calcium sulphate plus manganese. Observation temperature 110°C.* (*1*) *150,000 volts,* (*2*) *200,000 volts,* (*3*) *240,000 volts.*

is shown in Fig. 1. The effect upon the decay curves of this same material is shown in Fig. 2. It will be observed that exposures at higher voltages result in greater maximum brightness and slower rate of decay. Decay curves for calcite exposed at different voltages and observed at

two different temperatures are shown in Fig. 3. For both temperatures of observation there is less difference between the curves for the specimens exposed at 240,000 and 200,000 volts than for those exposed at 200,000 and 150,000.

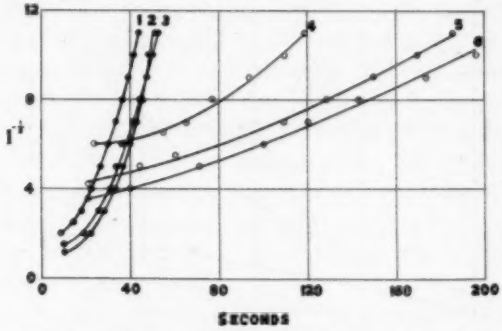

FIG. 3. *Decay curves of calcite exposed at different voltages and observed at different temperatures: (1) Voltage of exposure 150,000; temperature of observation 110°C. (2) Voltage of exposure 240,000; temperature of observation 110°C. (3) Voltage of exposure 200,000; temperature of observation 110°C. (4) Voltage of exposure 150,000; temperature of observation 65C°. (5) Voltage of exposure 200,000; temperature of observation 65°C. (6) Voltage of exposure 240,000; temperature of observation 65°C.*

PERSISTENCE OF THE EFFECT

Observations of the thermoluminescence of specimens exposed to cathode rays were made seven months after the exposure, the materials having remained at room temperature during the intervening time. At an observation temperature of 250°C, calcite, which had been exposed at a voltage of 200,000 and 240,000 for a period of 10 seconds showed bright luminescence which persisted for several minutes. A specimen of the same material exposed 50 seconds at 150,000 volts showed practically no thermoluminescence. Franklin fluorite exposed at 240,000 volts was still very bright when heated, that exposed at 200,000 less bright and that exposed at 150,000 still less bright. The difference in the persistence of the effect due to different voltages was not so great as in the case of calcite. Calcium sulphate plus manganese showed little difference in brightness between specimens which had been exposed at 240,000 and 200,000 volts but great difference between those exposed at 200,000 and 150,000 volts. The difference in the persistent effect due to different voltages was greatest in the case of calcite.

It appears that the speed of the incident electrons must come up to a certain value before the effect is permanent. Increasing the speed

beyond that value does not produce a great difference. It takes a higher voltage to produce a lasting effect upon calcite than upon the other materials. This may be due to a higher electrical resistance of calcite and a consequent greater difficulty in dislodging the electrons.

THE EFFECT OF VARYING THE TEMPERATURE OF EXPOSURE

The effect of cathode rays was found to vary greatly with the temperature of the specimen during exposure. In order to maintain the material at a constant temperature it was placed in a copper holder on the top of a copper rod 1 1/4 inches in diameter. The rod extended down into a wide-mouthed Dewar flask which contained liquid air, when a low temperature was desired, and water at 80°C, when a higher temperature was desired, the holder being about 1/4 inch above the surface of the liquid in each case. The actual temperature condition of the specimen after bombardment began is, of course, a matter of uncertainty.

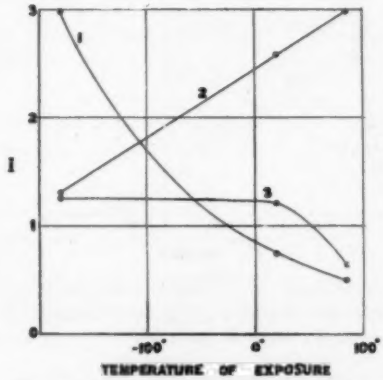

FIG. 4. *Maximum intensity after exposure at different temperatures. Observation temperature 160°C: (1) Calcite, (2) Calcium sulphate plus manganese, (3) Fluorite.*

Fig. 4 shows a decided difference in the behavior of the different specimens observed after exposure at three different temperatures.

Curve 1 shows that the maximum brightness of calcite is greatest after exposure at liquid air temperature. It was observed that during exposure at liquid air temperature the calcite was discolored a slate grey. As soon as the specimen warmed up slightly, the grey color disappeared and a brilliant cherry-red phosphorescence appeared which was all exhausted before the material could be taken from the holder. The fact that the thermoluminescence which appeared upon heating was more intense after exposure at liquid air temperature than after

exposure at room temperature indicates that the material absorbs much more energy at the lower temperature.

Calcium sulphate plus manganese absorbs more energy at higher temperatures than it does at lower, behaving just oppositely from calcite as is shown in Curve 2. The thermoluminescence observed at 160°C was greatest after exposure at 80°C and least after exposure at liquid air temperature. This material showed no luminescence under bombardment at the low temperature, but, while it was warming up to room temperature, it emitted a brilliant green light which appeared to be of the same color as the thermoluminescence obtained upon subsequent heating. The bombardment may have put the material into a less stable condition so that much of the energy was emitted at a lower temperature.

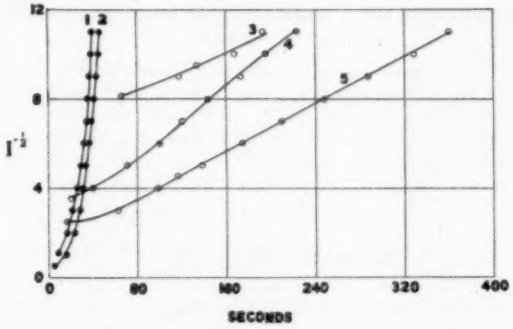

FIG. 5. *Decay curves of Calcite: (1) Exposed at room temperature, observed at 155°C: (2) Exposed at liquid air temperature, observed at 155°C, (3) Exposed at 80°C, observed at 60°C, (4) Exposed at room temperature, observed at 60°C, (5) Exposed at liquid air temperature, observed at 60°C.*

Franklin fluorite has the same maximum thermoluminescence after exposure at liquid air temperature and at room temperature but the brightness is much less after exposure at 80°C, as is shown in Curve 3. The material was a much more brilliant green under bombardment at liquid air temperature than at room temperature. The most intense part of this luminescence is exhausted while the specimen is warming up to room temperature. At 80°C the specimen does not absorb so much energy as at room temperature perhaps due to the emission of energy at this higher temperature during excitation.

Decay curves for calcite exposed at different temperatures are shown in Fig. 5. Curves 1 and 2 show that at a high temperature of observation the decay is rapid for material exposed at room and liquid air

temperatures and that there is little difference in the rate of decay. Curves 3, 4, and 5 show that at a low temperature of observation there is a great difference in the brightness and rate of decay of material exposed at different temperatures. The material exposed at liquid air temperature is much brighter and the thermoluminescence lasts longer than that of material exposed at room temperature, and the material exposed at 85°C is much dimmer than that exposed at room temperature. Apparently there are two distinct effects of cathode rays, one of which may be the ordinary phosphorescence which comes out at a low temperature and the other a thermoluminescence which requires a higher temperature for emission.

Fig. 6 shows the decay of calcium sulphate plus manganese exposed at 85°C and at liquid air temperature and observed at 110°C. The

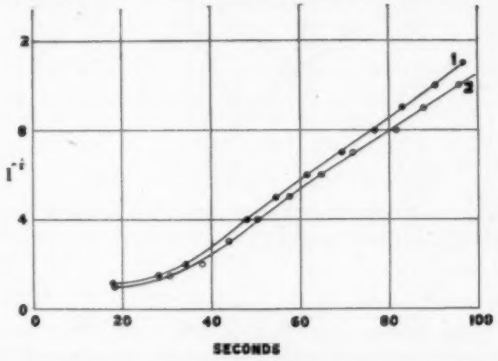

FIG. 6. *Effect of temperature of exposure upon decay of calcium sulphate plus manganese. Temperature of observation 110°C°: (1) Exposed at-180°C, (2) Exposed at 85°C.*

curves are very similar the only difference being one of brightness, the material exposed at 85°C being slightly brighter than that exposed at liquid air temperature. It is possible that the two processes observed in calcite occur in this specimen only when the exposure is made at low temperatures and that the first process takes place at a temperature below room temperature.

EFFECT OF VARYING TEMPERATURE OF OBSERVATION

The effect of varying the temperature of observation upon the maximum brightness of thermoluminescence is well known; the emission of energy takes place more rapidly at the higher temperature with greater brightness. Fig. 7 shows the maximum brightness of Franklin fluorite observed at different temperatures. Curve 1 is for the natural material not exposed to cathode rays; curve 2 for the heated

material exposed 10 seconds; curve 3 for the natural, unheated specimen exposed 10 seconds.

Fig. 8 shows the effect of varying the temperature of observation after different periods of exposure. For lower temperatures of observation the maximum brightness is practically the same for exposures of 1 second or more. The maximum at the higher temperatures is very dependent upon time of exposure. It will be observed from Fig. 5 that the shape of the decay curves of calcite changes greatly with a change in the temperature of observation, as has been previously found

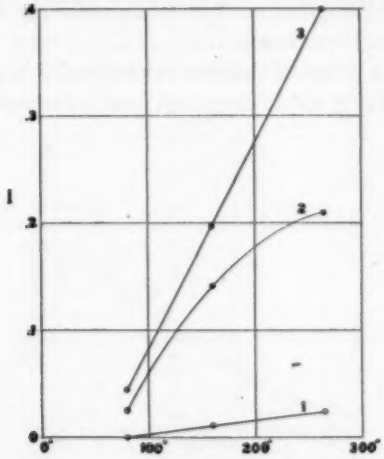

FIG. 7. *Effect of temperature of observation upon maximum brightness of Franklin fluorite: (1) Natural material, (2) Heated material, exposed 10 seconds, (3) Natural material, exposed 10 seconds.*

to be the case after exposure to x-rays.[5] The decay at the higher temperature is distinctly of the "vanishing type."

Fig. 9 shows that calcium sulphate plus manganese decays much more rapidly at higher than at lower temperatures. This material is very sensitive to temperature change, and the thermoluminescence may be observed at a lower temperature than that of the other specimens examined.

Fig. 10 shows the effect of allowing a specimen of calcium sulphate plus manganese to stand at room temperature after removal from the liquid air and before heating. The maximum brightness was obtained after an interval of about forty minutes. The maximum brightness of this same material after exposure to x-rays was obtained after leaving it for about fifty minutes at room temperature. This would indicate that there may be three energy states and that the cathode

or x-rays lift the electron from the normal state I to a state III and that at room temperature the change III to II takes place more rapidly

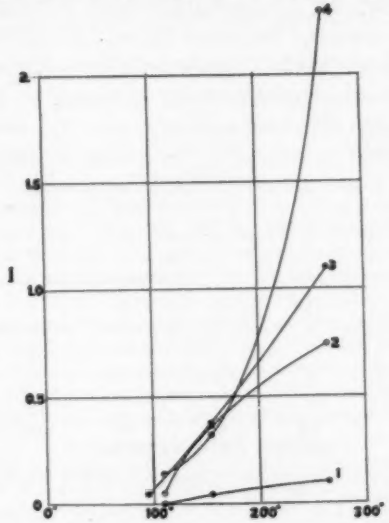

FIG. 8. *Effect of varying temperature of observation upon maximum brightness of calcium sulphate plus manganese: (1) Exposure 1/10 sec., (2) Exposure 1 sec., (3) Exposure 10 sec., (4) Exposure 25 sec.*

than that from II to I, the latter being the change which is accompanied by the luminescence. After standing at room temperature thus there

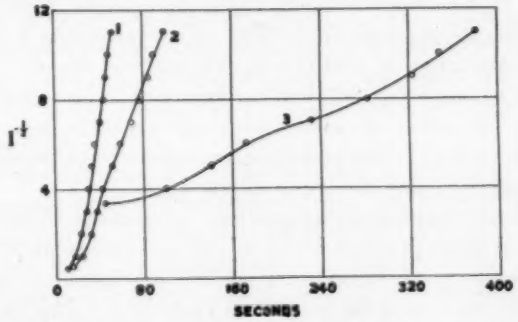

FIG. 9. *Effect of temperature of observation upon decay of calcium sulphate plus manganese: (1) Observation temperature, 145°C., (2) Observation temperature 110°C., (3) Observation temperature 65°C.*

would be more centers ready to change from II to I. This may explain also why, as shown in Fig. 4, the maximum brightness of the calcium

sulphate plus manganese was greater after exposure at the higher temperature.

The effects produced by exposure to high voltage cathode rays are similar to those produced by x-rays but with some marked differences. Materials which are made thermoluminescent by exposure to x-rays are also made thermoluminescent by exposure to cathode rays. As has previously been observed, exposure to x-rays results in a thermoluminescence which, upon heating the specimen, comes up quickly and

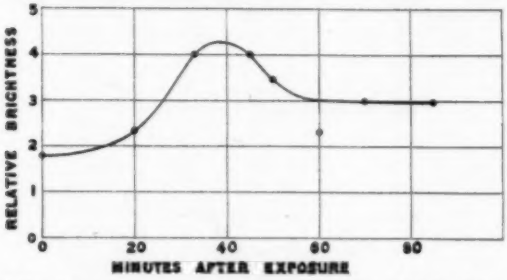

FIG. 10. *Relation between maximum brightness and the time elapsing between removal from liquid air and the application of the heat.*

lasts for a very short time followed by another luminescence sometimes of the same color, sometimes of a different color, which comes up more slowly and lasts longer. Only the more persistent of these emissions is conspicuous after exposure to cathode rays. There are indications that the other one is present but it is relatively much less intense. It required a higher temperature to excite the material after exposure to x-rays than after exposure to cathode rays. Saturation is produced in a much shorter time by exposure to cathode rays than by exposure to x-rays, 10 seconds being sufficient for the cathode rays and 60 minutes for x-rays. The effect of cathode rays appears to be more permanent than that of x-rays. The high voltage cathode rays produce a much more lasting effect than the lower voltage ones.

It is difficult to make any satisfactory comparison of the effects due to x-rays and cathode rays because of the uncertainty as to the energy imparted to the material by each method of exposure. Both x-rays and cathode rays must be present in each case. The fact that the luminescence produced by cathode rays is so much more intense than that produced by any other source of excitation indicates that the electrons are in every case the direct cause of the luminescence.

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INSTRUMENT SECTION

A DIRECT-CURRENT AMPLIFIER FOR MEASURING SMALL CURRENTS

By J. M. Eglin

ABSTRACT

A direct-current amplifier consisting essentially of a Wheatstone bridge, having the amplifying tube in one arm and a balancing tube in another, has been described by P. I. Wold and by C. E. Wynn-Williams. This circuit has now been developed to give a constant amplification for currents in either direction up to 10,000 times the lowest measurable value. The amplification and the lowest measurable current are alterable together by changing the resistance introduced between the grid and filament of the amplifying tube. With tubes of high insulation, the amplification can be made as large as 10^6; and the measurable current as low as 10^{-14} ampere. Some improvements of the circuit are: (1) the insertion of a resistance in series with the tube in one arm of the bridge to "compensate" for variations in plate and grid battery voltages; (2) the suspension of the tubes to protect them from mechanical vibrations; (3) the use of tubes with pure tungsten filaments to avoid changes in contact potentials, and with plates enclosing the filaments completely to lower the effects of wall charges. In a "null" method of using the circuit the values of the grid resistance and an auxiliary potential introduced in the grid-filament circuit are sufficient to determine the measured current.

One of the methods available for the measurement of small direct currents consists of amplifying them and measuring the amplified values. Vacuum tube circuits are commonly used for this purpose. A circuit which appears to offer special promise was developed by P. I. Wold of the Western Electric Company[1] about ten years ago. It has also been employed more recently by Wynn-Williams.[2,3] The present article deals with further development of this circuit. The substance of the paper was presented before a recent meeting of the American Physical Society.[4]

The arrangement used by Wynn-Williams is shown in Fig. 1. It consisted essentially of a Wheatstone Bridge, two arms of which were resistances R_1 and R_2, and two were vacuum tubes T_1 and T_2. The "B" battery for the tubes, E_b, also served as the battery of the bridge; and the "A" battery was common to both tubes. The input was introduced

[1] P. I. Wold, U. S. Patent, No. 1,232,879; 1916–1917.
[2] C. E. Wynn-Williams, Proc. Camb. Phil. Soc., *23*, p. 810; 1927
[3] C. E. Wynn-Williams, Phil. Mag., *6*, p. 324; 1928.
[4] J. M. Eglin, Phys. Rev., *33*, p. 113; 1929.

between the grid and the filament of one tube (T_1 in this case); it changed the voltage of the grid with respect to the filament, and thereby the effective resistance of T_1 and hence the balance of the bridge. The output or amplified current was the unbalance produced, and was read on the meter M.

Wynn-Williams[2] has pointed out the advantages of this circuit over the ordinary single-tube type of amplifier. The main advantage is that the two tubes react similarly to variations of filament and plate batteries, and therefore, due to the bridge arrangement, reduce the importance of such variations.

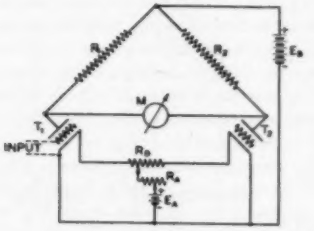


FIG. 1. *The Wynn-Williams circuit.*

As noted above, the circuit Wynn-Williams chose to employ connected one terminal of the input to the grid of one tube, and the other terminal to the filament of that tube. This essentially made the leakage between grid and filament (not shown in the figure) a portion of the input circuit. We have preferred (as did Wold[1]) to have a definite and known resistance introduced by the amplifier into the input circuit, especially since we can, at the same time, set the grid at a definite negative voltage. This can be done by the use of a grid resistance R_{g1} and a grid battery E_c as shown in Fig. 2. The same grid battery and a similar resistance R_{g2} are used with the other tube.

In this arrangement, one can see more clearly the nature of the function performed by the vacuum tube T_1. It takes a voltage obtained from a small current passing through a high resistance in the grid circuit, and effectively transfers this voltage, (somewhat increased due to the amplifying action of the tube) to the plate circuit. In this latter circuit, which has a much lower resistance, a much larger current is thereby produced.

[1] Loc. cit.
[2] Loc. cit.

The grid battery E_c is made sufficiently large so that the electron current to the grid is negligible. Otherwise we would be forced to take into consideration the changes in the grid current produced by changes in the grid voltage; and our amplifier would depend for the constancy and reliability of its amplification factor (the ratio of amplified to unamplified current) upon a grid current-grid voltage characteristic which is never very steady.[5]

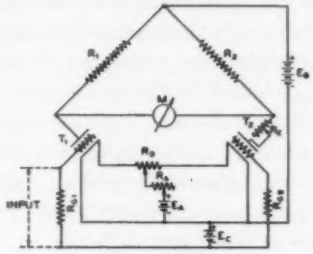


FIG. 2. *The circuit with grid battery, grid resistances, and plate compensation.*

With the grids at these negative potentials, the amplification factor of the bridge is also the same for both positive and negative currents, and is quite constant up to output currents which are an appreciable fraction of the space current of the tubes. The variation at these high currents is due only to the departure from linearity of the plate current —grid voltage characteristics of the tubes.

The formula we have deduced for the amplified current through the meter M is

$$i_m = \frac{\mu \Delta E_g \dfrac{R_1}{R_1+R_{01}}}{R_m + \dfrac{R_1 R_{01}}{R_1+R_{01}} + \dfrac{R_2 R_{02}}{R_2+R_{02}}}$$

where μ is the amplification factor of tube T_1; ΔE_g is the change in grid voltage of tube T_1 due to the input current; R_1 and R_2 are the resistances in the bridge; R_{01} and R_{02} are the alternating current resistances $(\Delta E_b)/(\Delta I_b)$ of the tubes T_1 and T_2; and R_m is the resistance of the meter.

This formula reduces to the one given by Wynn-Williams[1] if we

[5] P. Lertes, Zeit. f. tech. Phys., *4*, p. 317; 1923.

[1] Loc. cit.

assume that $R_1 = R_2$, $R_{01} = R_{02}$, provided that we interpret his X as the alternating current resistance of the tubes. (He actually used X for the direct current resistance). It also reduces to the same form as that obtained by Razek and Mulder[6] in a more recent consideration of the circuit. ΔE_g, as noted above, is the change in the effective voltage of the grid of T_1 due to the input current i_u. If we wish a high amplification we must have a large ΔE_g for a given i_u. To get the highest amplification possible the grid resistance through which we pass i_u should be of the same order as the leakage resistance between the tube terminals or the external leakage resistance between the input leads.

Actually $\Delta E_g = i_u R_g$, where i_u is the input current after the amplifier is inserted into the input circuit, and R_g is a sort of effective resistance between grid and filament. If the leakage resistances between grid and filament and between plate and grid of tube T_1 are large compared to R_{g1}, then $R_g = R_{g1}$ is a sufficiently good approximation. If the leakage resistances are of the same order as R_{g1}, then R_g becomes a much more complicated expression.

Some values of the grid resistance R_{g1} used with our bridge are given in the table, together with the current amplification $(i_m)/(i_u)$ and the lowest measurable current. The latter is defined for the present purpose as that current at which the probable error of a single reading is of the order of a few tenths of the reading. Thus to measure currents of the order of 10^{-12} ampere, the grid resistance was 100 megohms For 10^{-14} ampere, it was 10,000 megohms. As a current-measuring instrument, the amplifier therefore acts like an ammeter of very high resistance. When introduced into the input circuit, it will perhaps alter the input current. If the original current is to be measurable, the *altered* current must, in passing through the resistance, produce a potential difference between the ends of 4×10^{-5} volt or more.

TABLE 1. *Some values for the circuit and tubes used*

Input resistance R_{g1} in megohms	Current amplification	Lowest measurable current in amperes
1	180	10^{-10}
100	18000	10^{-12}
10000	900000	10^{-14}

[6] J. Razek and P. J. Mulder, Phys. Rev. *33*, p. 284, 1929.

There are variations of many items with which one must deal in a set of this sort. It is of course very desirable that the balance or zero of the amplifier shall not vary during the time of taking a reading. The resistances of the tubes concerned are altered if the values of the voltages impressed between their elements change; and thus variations in the balance occur. Hence, if the voltages furnished by the tube batteries vary unavoidably, the operating potentials of the tube elements should always change together in such a way that the balance of the bridge is not affected. To maintain this condition we must satisfy certain requirements with respect to the differentials of the currents involved which can be experimentally fulfilled by introducing auxiliary resistances of the proper value.

Thus, for fluctuations in the filament battery voltage, the condition that the bridge balance shall remain steady was deduced by Wynn-Williams[1]. It is

$$\frac{\frac{\partial I_{b1}}{\partial E_a}}{I_{b1}} = \frac{\frac{\partial I_{b2}}{\partial E_a}}{I_{b2}}$$

where I_{b1} and I_{b2} are the plate currents of tubes T_1 and T_2 respectively, and E_a is the voltage of the filament battery. To produce this condition experimentally Wynn-Williams introduced a "compensating" resistance R_D (Fig. 1), the function of which was to regulate the relative current to each filament. We were able to compensate in this way so well that a change of .05 volt in the 4.25 volts required for the filament battery voltage produced only a change of 5×10^{-8} ampere in i_m. The tubes should be run on the filament saturation portion of the characteristic, so that changes in filament voltage are relatively unimportant as regards the plate current. There is an additional effect which does not seem to have been considered by Wynn-Williams in that changes in filament voltage may also alter the effective grid voltage with respect to different parts of the filament.

For fluctuations in the plate battery voltage, the condition is similar to that given above, but reduces to $R_{01}/R_{T1} = R_{02}/R_{T2}$ where R_{01} and R_{02} are the alternating current resistances $(\Delta E_b/\Delta I_b)$ of the bridge arms containing T_1 and T_2 and R_{T1} and R_{T2} are the direct current resistances (E_b/I_b) of the same arms. These values were, in the circuit

[1] Loc. cit.

of Wynn-Williams, fixed by the tubes themselves, and would not be alike. They may, however, be altered by inserting a resistance R_E (Fig. 2) in series with the tube. Thus if $R_{01}/R_{T1} > R_{02}/R_{T2}$, insertion of R_E will change R_{02}/R_{T2} to $R_{02}+R_E/R_{T2}+R_E$ (as far as the bridge is concerned) which is larger than R_{02}/R_{T2}, since from the characteristic properties of the tube R_{02} is less than R_{T2}. We can if we wish make the fraction equal to R_{01}/R_{T1} in this way. If this is done small changes in the plate battery at the operating point are compensated. We have found it posssible in a particular case to compensate for changes in E_b so that a change of one volt (with $E_b = 120$) would produce only an alteration of 5×10^{-9} ampere in the balance current. This general method has been used for alternating current circuits by Mohr[7] of the Bell Telephone Laboratories.

For changes in the grid battery potentials it is possible to do much the same sort of thing. The criterion reduces to

$$\mu_1 k_1 \frac{R_1}{R_{01}} = \mu_2 k_2 \frac{R_2}{R_{02}} \quad \text{where} \quad k_1 = \frac{1}{1+\dfrac{R_{g1}}{R_{L1}}+\dfrac{R_{g1}}{R_{L'1}}}$$

$$k_2 = \frac{1}{1+\dfrac{R_{g2}}{R_{L2}}+\dfrac{R_{g2}}{R_{L'2}}}$$

in which μ_1 and μ_2 are amplification factors of tubes T_1 and T_2, and R_{L1} and $R_L'_1$ are the grid-to-filament and plate-to-grid leakage resistances of tube T_1, and R_{L2} and $R_L'_2$ and the corresponding quantities for tube T_2. If we wish to compensate for variations in E_c we must select a proper R_g for tube T_2 or T_1 as the case may be. If any compensation of this nature is to be made, R_{g1} or R_{g2} must be comparable in size with the grid-to-filament or plate-to-grid leakage resistances. It is not easy to obtain a large supply of high resistances of desired values or to get an adjustable high resistance which will be very satisfactory. However, this sort of compensation has been used experimentally by Razek and Mulder[6]. An easier way of avoiding the difficulty is to get tubes of approximately equal amplification factors for T_1 and T_2—which is not hard if the other characteristics do not need to fit—and use the plate compensation previously described. If R_g is smaller then

[7] F. Mohr, U. S. Patent, 1,636,146; 1922–1927.
[6] Loc. cit.

R_{L1} and $R_L'_1$, and R_{g2} is similarly related to R_{L2} and $R_L'_2$, then k_1 will be approximately equal to k_2 and the compensation relation will be fulfilled. Variations of the grid battery voltage are small, in any case, since there is practically no drain on the grid battery.

The usual precautions taken to make the whole electrical circuit secure and free from external disturbances are very necessary. All connections should be soldered and free from thermoelectric voltages.[8] Contacts to the resistances should be of only the highest grade, and the resistances themselves should have ample carrying capacity and small temperature coefficients. Wynn-Williams in his filament compensation circuit passed a slider along a wire to vary the relative resistance in the filament circuits of the two tubes. We have replaced this by a movable binding post which can be tightened on the wire at the point desired.

If external mechanical vibrations are at all troublesome they must be avoided by a special mounting for the tubes[9]. For this we have tried many schemes. A small Julius suspension holding both tubes has proved by far the best. We constructed a reduced model of one such as recommended by Johnsrud[10]; the same sort of a mounting has also been used by Johnson[11] in his work on the "flicker" and "shot" effects. When inserted in this, the tubes were insensitive to practically all external vibrations including sound waves.

Shielding against electrical disturbances, is quite important—especially for the grid-circuit portion of the apparatus. This has been emphasized by du Prel[9] and by Wynn-Williams in his second article.[3] That part of the input apparatus from which the current to be measured is taken should be well shielded where it is not at a definite potential with respect to the ground.

The tubes used should be well degassed. Production of high amplification with tubes containing a small amount of gas is possible[12]. It is not to be recommended in general because of the change of gas pressure inevitably taking place. Alterations of the constants of the circuit (K in the paper mentioned) result, and the correspondingly more pronounced variations in the amplification necessitate frequent recali-

[8] R. Jaeger and Kussmann, Phys. Zeit., *28*, p. 645; 1927.
[9] G. duPrel, Ann. d. Phys., *70*, p. 199; 1923.
[10] A. L. Johnsrud, J. O. S. A., *10*, p. 609; 1925.
[11] J. B. Johnson, Phys. Rev., *26*, p. 71; 1925.
[12] P. J. Mulder and J. Razek, Phys. Rev. 33, p. 284; 1929.

brations. The gas present may also affect filament activitites, contact potential differences, and wall charges. In the past all sorts of attempts have been made to avoid ionization of residual gas [8, 9, 13, 14, 15]; and it is well to insure its absence as much as possible.

It is desirable to have tubes of steady characteristics such as those with pure tungsten filaments. The use of such filaments helps together with the absence of gas noted in the preceding paragraph, to keep down variations in the contact potential differences between the grid and the filament, which may with other types of filament or the presence of gas be very bothersome. Contact potentials may in some tubes vary from one value to another, differing perhaps by a fraction of a volt or more, for quite a period of time after each change in the operating voltages. When a potential difference of 10^{-5} volt is involved in making the measurements, a tenth of a volt is comparatively large. With tungsten filaments and the absence of gas, the variations are, however, not as large as under other conditions.

Wall charges are also a source of trouble. The walls being of glass, these charges leak off very slowly if the plate voltage on the tube is changed. The charges have a grid action on the space current, and this will be quite different for different tubes depending upon the form of the tube, the distribution of the charges on the glass, etc. The effects would be least noticeable in a tube in which the plates shield the filament from the influence of the walls, i.e., enclose it as much as possible.

If it is desired to measure very small currents it is also necessary, as explained above, to have high insulation between the tube electrodes to permit the use of high external grid resistances. We obtained this by using unbased tubes with the grid lead brought out through the upper part of the bulb.

There were still left after taking precautions (perhaps not extensive enough) along the lines mentioned above, two types of unsteadiness of the zero, or balance point of the meter used in the output circuit. One was a gradual drift of the zero in a constant direction after throwing on the voltage– also noted by Wynn-Willaims[2]—which

[8] Loc. cit.
[9] Loc. cit.
[13] H. Greinacher, Zs. f. Phys., *23*, p. 379; 1924.
[14] H. Rosenberg, Naturwiss., *9*, p. 359, p. 381; 1921.
[15] K. W. Hausser, R. Jaeger, and W. Vahle, Wiss. Ver. Siemens Konz., *2*, p. 325; 1922.
[2] Loc. cit.

amounted in our set to .04 volt effective change in grid voltage of T_1 (or 4×10^{-10} ampere unamplified current with the 100 megohm resistance) in the first 20 minutes, and thereafter a small and decreasing change of the order of .01 volt in 20 minutes. Since readings can be taken with a meter of short period, this drift is not bothersome after the first few minutes. The other residual variations in the zero were erratic vibrations of the order of 2×10^{-5} volt effective grid voltage (or up to 4×10^{-13} ampere unamplified current with the 100 megohm resistance) around the position of balance or of the reading. These persisted no mattter how long the set had been running and may have been due to lack of sufficient shielding. Their order of magnitude seemed to be larger than the fortuitous (thermal agitation) disturbances to be expected[16,17].

We have not tried to employ regeneration with the circuit, though such an effect could be worked out if another stage were added to the amplifier. Regeneration of this sort has been used in the past by Hartley[18], Jaeger and Scheffers[19] and by Carwile[20].

In connection with this circuit we have used a null method similar to one which we later found had also been proposed by Wold[1]. The

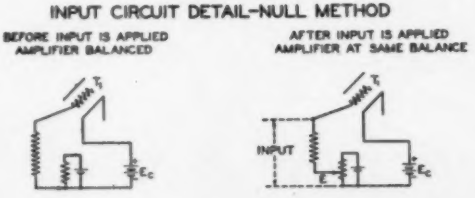


FIG. 3. *The input circuit as in the null method.*

off-balance is produced, as noted above, by putting the current input to the amplifier through the grid resistance. (Fig. 3 shows the input portion of the circuit.) We can restore the bridge to balance by introducing an equal and opposite potential in series with this grid re-

[16] G. Ising, Phil. Mag., *1*, p. 827; 1926.
[17] F. Zernike, Zs. f. Phys., *40*, p. 633; 1926.
[18] R. V. L. Hartley, U. S. Patent, 1,218,650; 1914–1917.
[19] Jaeger and Scheffers, Wiss. Ver. Siemens Konz., *4*, 233; 1925.
[20] T. B. Carwile, Phys. Rev., *33*, p. 284; 1929.
[1] Loc. cit.

sistance between the filament and the grid. This can be done with a potentiometer and the voltage E introduced can be read on a voltmeter. The necessity for calibration of the circuit is thereby avoided, since the current to be measured is simply $i_u = E/R_{g1}$.

BELL TELEPHONE LABORATORIES,
NEW YORK, N. Y.
FEB. 6, 1929.

A SPHERICAL HIGH-PRESSURE IONIZATION CHAMBER

By James W. Broxon

Work done by the writer[1] and other experimenters[2] on the natural ionization in gases at high pressures, has shown the desirability of extending the pressure range considerably above the maximum pressure previously attained, about 1000 lbs. per sq. in. The construction of a new ionization chamber for use at pressures in the neighborhood of 2500 lbs. per sq. in. and of dimensions comparable with those of the one formerly used, presented new difficulties which have been overcome in a satisfactory manner.

As shown by photographs in the papers[2] (by Swann and Downey) mentioned, the chamber previously used consisted of two hemispheres separately cast and machined, and held together by bolts through flanges. In order to facilitate calculations and make possible direct comparisons with previous results, it was desired to make the interior of the new chamber of the same shape and size as the old one. At the proposed pressures, however, the flanges would have to be very thick and the bolts correspondingly long. Under these circumstances, the bolts would stretch sufficiently to increase greatly the difficulty of maintaining a gas-tight junction. Also, if more than one row of bolts were used, the increased size would considerably increase the difficulty of shielding from external radiations. Moreover, the apparatus so constructed would be difficult to manipulate and expensive to construct.

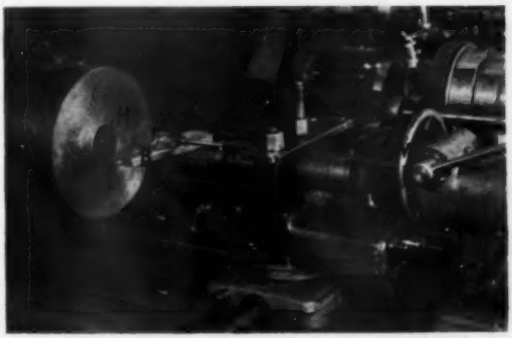


Fig. 1. *Spherical high-pressure ionization chamber.*

[1] J. W. Broxon, Phys. Rev., *27*, p. 542; 1926.

[2] K. M. Downey, Phys. Rev., *16*, p. 420; 1920; H. F. Fruth, Phys. Rev., *22*, p. 109; 1923; W. F. G. Swann, Jour Frank. Inst., *205*, p. 767; 1928.

In view of these considerations, it was decided to excavate a spherical cavity from a solid cylinder of wrought steel, the small entrance to be closed by a plug with tongue, groove and gasket. The method of procedure will be obvious upon inspection of the photograph. The cylinder was 15 1/8 in. in diameter and 17 3/8 in. long. A cylindrical hole, *H*, 3 11/16 in. in diameter and extending to within about an inch of the far side of the desired spherical cavity, was first machined. This was then enlarged to a diameter of 7 5/8 in. within the region of the spherical cavity, by means of an ordinary boring tool. The tool shown in the photograph was next employed. The bar, *B*, could be rotated about a central, one-inch pin fastened to bar, *C*. The rotation was effected by means of a 66-tooth beveled gear fastened to *B*, and a 22-tooth pinion fastened to the rod leading from the wheel, *W*. These gears were covered with a sheet of copper in order to exclude the steel shavings thrown over them, and hence are not shown in the photograph. *B* was initially lined up with *C* and adjusted until the point of intersection of the axis of rotation of *B* and the line connecting the tips of the tools, *T*, coincided with the center of the desired spherical cavity. Upon rotation of *W* by means of a rod and weight not shown, the tool points moved along a circle with its center at the center of the cavity. The gravity feed was controlled by the tool clearance, and the cutting proceeded smoothly with no chattering. After each cut, the tool was removed and the tool points extended. Actually, two different bars, *B*, were used at different stages of the excavation, the first 7 1/2 in. long. and the second 10 1/2 in. long.

In the above manner, a spherical cavity of 11 23/32 in. diameter was excavated, the cylindrical inlet being 3 11/16 in. in diameter and 3 7/16 in. long when the excavation was completed. After some polishing, the diameter of the sperical cavity was found to vary less than 0.002 in.

The new ionization chamber should be able to sustain a force ten times as great as the old one at its weakest cross-section. It is more compact, is made of a more desirable material, and is much more easily manipulated than the old one. Moreover, the cost was only about one-half as much as it would have been with the former design.

The writer wishes to express his appreciation of the advice of Mr. L. J. Brunton, instructor in Mechanical Engineering, and Mr. M. M. Eaton, departmental mechanician, the latter of whom did the actual constructing of the apparatus.

HALE PHYSICAL LABORATORY,
UNIVERSITY OF COLORADO,
BOULDER, COLORADO,
JANUARY 18, 1929.

BEHAVIOR AT IMPACT OF JETS OF HIGHLY COMPRESSED AIR

By John Zeleny

Those working with jets of highly compressed gases such as are used in air liquefiers may be interested in the following observation on the behavior of such jets when they impinge against a solid surface. In the liquefier employed in this laboratory the liquefied air is caught as is usual in a receptacle situated immediately below the openings from which the air compressed initially to 200 atmospheres issues horizontally. A question arose as to whether the jets of unliquefied air coming from these openings produce any splashing of the air already liquefied and caught in the receptacle. If the liquid were dashed about and some of it thrown up into the copper coils, a loss of efficiency would result

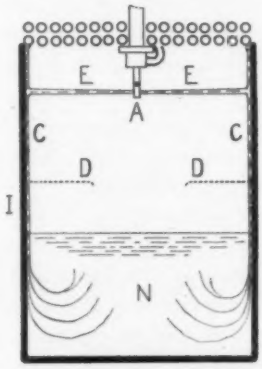


Fig. 1. *Vessel below air liquefier coils.*

because of an increased loss by heat conduction produced by the establishment of a greater temperature gradient than need be in parts of the coils some distance above the reduction valve.

The liquefier coils were removed from their packing and set up in the open air. A glass jar, *I* in Fig. 1, 12.5 cm in diameter and 18 cm high, was partly filled with water, *N*, and placed as shown. The jets of air, *EE*, coming from the openings of the nozzle *A* struck the walls of the glass vessel normally. When the air compressor was started and the air pressure gradually rose, the water in the glass vessel remained

very quiet until the pressure was approaching the working pressure when all at once the water was all dashed out of the vessel. The action was the same when the cylindrical container, *I*, was replaced by one in the form of an inverted cone having a total angle of 60°.

At the critical point it seemed as though the jets flattened out against the glass wall and spread in all directions as represented by *CC* in Fig. 1. The part of the air going downward cut into the water as would a knife and when it turned upward it simply forced all of the water out ahead of itself. The suddenness with which this occurred and the completeness with which the water was removed were most surprising. The action was easily overcome by introducing a baffle ring, shown by the broken lines *DD*, which prevented the downward spread of the air streams. Such a baffle was placed into the collecting vessel of the liquefier to avoid the possibility of any such commotion of the liquefied air, but no data are available as to whether any increase of efficiency has resulted.

SLOANE LABORATORY,
YALE UNIVERSITY,
MARCH 21, 1929.

NOTE ON HIGH VOLTAGE LEYDEN JARS

By Philip P. Quayle

In general laboratory work, one occasionally has need of condensers of moderate capacity capable of withstanding potentials of the order of 10^5 volts. It is sometimes desirable to charge such condensers to a high potential by some such means as a transformer and Coolidge tube, and then to hold this charge for a short interval before discharging. In this event, the dielectric strength of the condensers must not only be able to resist rupture due to very high peak voltages but the leakage rate must be low.

The method generally resorted to where commercial equipment is not available is the conventional stack of window glass plates and lead foil sheets. Such an arrangement has many objectionable features. Ordinary window glass which is a sodium-calcium silicate usually has a very high rate of leakage, hence, although the glass may not rupture the charge is rapidly dissipated by surface conduction, ionization of surrounding air, etc. Furthermore, due to the intense electric field surrounding such apparatus, dust particles are forcibly drawn into every crevice of the condenser. In this manner the border betweeen the lead foil sheet and the edge of the glass plates is soon filled with dust, which is more or less conducting, with the result that leakage is very greatly increased. Dust so located is almost inaccessible, necessitating complete dismounting of the condenser in order to clean it properly. If this type of condenser is screened in some way these faults are reduced in magnitude but they are not eliminated.

A very thin layer of foreign material will generally serve as a conducting path along which the condenser will "break down" and once such a discharge occurs it takes place even more readily thereafter.

It is the purpose of this note to describe briefly how one may construct condensers possessing the required insulation resistance and being generally free from the faults just enumerated.

The type of Leyden jar available from the various manufacturers of such equipment is altogether too small for the purpose in question. However, a battery consisting of six jars, such as that shown in Fig. 1, have been in constant use at this laboratory for over a year with excellent results.

The jar shown in Fig. 1 is approximately 38 cm high, 11.5 cm in diameter and the lead foil coatings are 20 cm high. The approximate capacity of this jar is 0.002 microfarads, thus giving a total capacity for the battery of six jars in parallel of 0.012 microfarads. The glass is Pyrex of an average thickness of 3.5 to 4.0 mm and was furnished at

FIG. 1. *Pyrex leyden jar.*

the author's request by the Central Scientific Company of Chicago. The insulation of these jars is such as not to require cleaning oftener than once in five or six months and when necessary this is readily accomplished in a few minutes in that it is only necessary to remove the dust with a cloth.

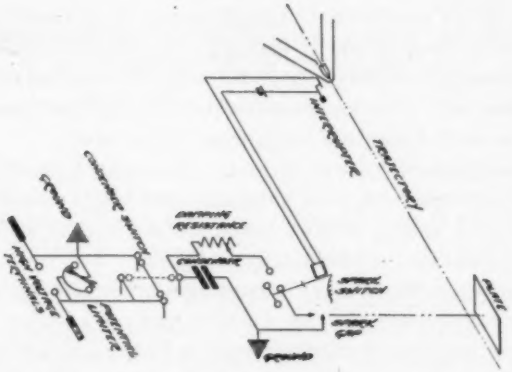


FIG. 2. *Schematic wiring diagram spark photography apparatus.*

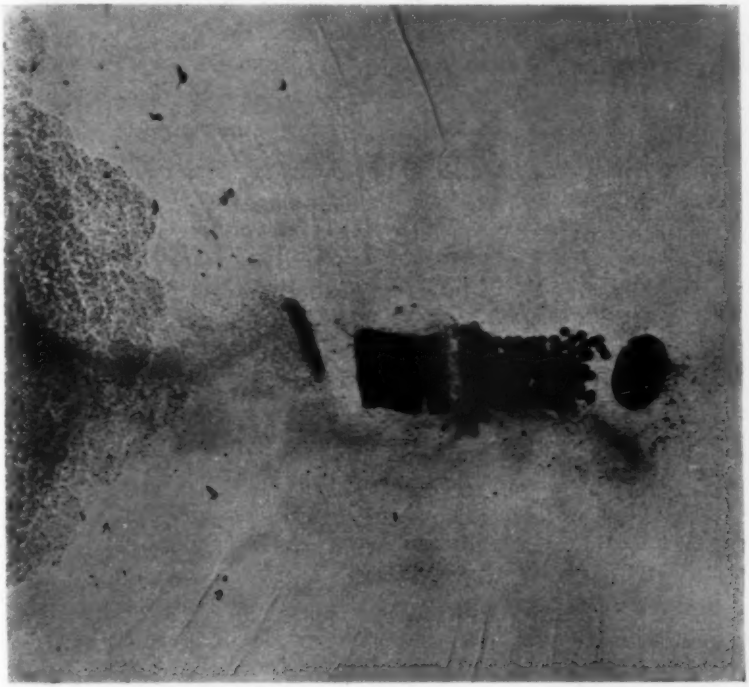


FIG. 3. *Spark photograph of a charge of shot near the muzzle of the gun.*

Leyden jars by commercial manufacturers, both domestic and foreign, will often break down by leakage or rupture if occasionally subjected to potentials of the order of 10^5 volts for a period of four to six months. However, in the fourteen months that the Pyrex jars have been in service no such difficulties have been encountered.

Some idea of the intensity of the spark obtainable from such a battery of suitable condensers may be obtained from Fig. 3 which is a spark photograph of a charge of shot taken by the author using a circuit such as that indicated schematically in Fig. 2.

Such condensers have many advantages; they are very light and hence easily transported, they are quite rugged and with any reasonable care are of sufficient mechanical strength to last practically indefinitely. Last, but by no means least, they are quite inexpensive.

THE PETERS CARTRIDGE COMPANY,
KING'S MILLS, OHIO.

EXPERIMENTAL METHODS OF STUDYING STRESS DISTRIBUTIONS

By R. V. Baud

ABSTRACT

The photoelastic, strain measurement, analogy and Lueder's line methods of studying stress distributions are discussed in this paper. It is concluded that these methods make extensive use of optics and it is suggested that this interesting field be given some attention by the optical industry.

Introduction: When the shapes of engineering structures deviate much from designs already known as safe from experience it is felt that there should be some means of confirmation for the approximate analytical solution. Recourse is made in such cases to experiments. The scope of this paper is to make a survey of the available methods and also to show to what extent optics are involved.

Notations:

p, q = Principal stresses, See Appendix

e_p, e_q = Principal unit strains; e = Unit strain in lateral direction

E = Modulus of elasticity; $G = E/2(1+\nu)$ = Modulus of rigidity

ν = Poisson's ratio (1/m)

t = Thickness of model

a, b, c, = Optical constants

p_x, q_y = Normal components of element oriented with sides parallel to x, y axes

s = Shear stress acting on element oriented with sides parallel to x, y axes

s_{ult} = Absolute largest shear stress acting on element, see Appendix

p_{xo}, q_{yo} = Normal components of elements, oriented parallel to x, y axes, from which integration in accordance with equation 5 and 6 is started

p_o, q_o = Principal stresses of elements, from which integration in accordance with equation 7 and 8 is started

ϕ = Angle between the direction of the principal stress p and the positive direction of the x-axis. Locus of points having equal angle ϕ are called isoclinics. ϕ is used as parameter of the isoclinics

ϕ_0 = Parameter of isoclinic at which integration is started

$d\phi, \Delta\phi$ = Increments in parameter ϕ.

$\psi(\psi')$ = Angle through which the direction of the p (q) stress has to be rotated counterclockwise in order to make it tangent to the isoclinic.

Δy = Intercept between two near isoclinics, whose parameter differ by $\Delta\phi$ (Δy to be measured perpendicular to the path of integration).

ds_1 (ds_2) = Increment in path of integration along the p (q) principal stress direction lines.

Photoelastic Method: The origin of the word photo is the Greek word phot or phos, i.e. light. Any experimental method using light for the determination of stress or strain may therefore be classed as a photoelastic method. However, it is customary to restrict the name to experimental methods based upon the phenomenon, that isotropic materials become aeolotropic under certain conditions, such as produced, for instance, by mechanical deformations.

On the basis of this phenomenon and by means of an interferometer, the principal stresses p and q can be obtained in a flat model loaded in its plane, by measuring the time-phase separations R_1 and R_2 of the refracted rays with respect to the original ray. R_1 and R_2 are connected with the principal stresses p and q by equations (1) and (2):

$$R_1 = atp + btq \tag{1}$$

$$R_2 = btp + atq. \tag{2}$$

By using crossed polarizing prisms and a compensator, the time-phase separation $R = R_1 - R_2$ between the two refracted rays is obtained, which is proportional to the stress difference $p - q$, since:

$$R = c(p-q)t \tag{3}$$

The optical constants a and b of equation (1) and (2) are related to the constant c of equation (3) by the simple form

$$a - b = c. \tag{4}$$

A method has been worked out by the writer which allows the evaluation of the stress difference p-q in a very simple manner[1].

[1] Baud, Further Development in Photoelasticity, see page 422 of this issue of the Journal.

The angle ϕ is established by observing on a screen the directional bands, i. e. isoclinics, which are obtained by locating the loaded model between two crossed polarizing prisms and by rotating either the model or the polarizing prisms.

The stress distribution is completely determined, when p, q from equation (1) and (2) and ϕ, or p-q from equation (3) and ϕ are known for every point of the model.[2] The mathematical process to separate p and q from p-q, which has been suggested by Clerk Maxwell[3] in the early history of photoelasticity, has proved unworkable, since it involves mathematics much too complicated for engineering use. Simpler methods have since been proposed by Coker and Filon.[4] They are based upon the differential equations obtained by formulating the equilibrium condition of an element. The first method consists of making a network of straight lines parallel to x and y axes and to integrate graphically along these straight lines in accordance with equations (5) and (6)

$$p_x = -\int_0^x \frac{ds}{dy}dx + p_{x0} \tag{5}$$

$$q_y = -\int_0^y \frac{ds}{dx}dy + q_{y0} \tag{6}$$

in which s is related to the stress difference p-q from photoelastic tests as expressed by equation (16), see appendix. Equation (5) and (6) taken in combination with equation (15) allow the principal stresses p and q to be computed. The fact that the difference between these stresses after the separation must be equal to the same difference before separation offers a valuable means of confirming the experimental and mathematical work. The same holds for the separation by using

[2] It is also necessary to know for a few points their normal components p_{x0} and q_{y0}, or principal stresses p_0 and q_0 respectively. Since at a free boundary either $q=0$, therefore, $p-q_0=p_{x0}=p_0$, or $p=0$, therefore, $p-q=-q_{y0}=-q_0$ such a boundary is a convenient place from which the integration may be started, in fact, it will be as a rule the only place for which p_0, p_{x0}, q_0 and q_{y0} are directly obtained from photoelastic tests with the equipment ordinarily used, consisting of two polarizing prisms, lenses, etc.

[3] Maxwell, On the Equilibrium of Elastic Solids, Trans. Roy. Soc. Edin., 1850, and Collected Papers.

[4] Filon, On the Graphical Determination of Stress from Photoelastic Observations, Coker, The Determination of Stresses at a Point in a Plate; Engineering, p. 511, 512; October 19, 1923.

the second method, which consists of mapping lines, such that their tangents represent the principal stress directions and to integrate graphically along these so-called principal stress direction lines in accordance with equation (7) and (8):

$$p = p_0 + \int_{\phi_0}^{\phi} (p-q) \operatorname{cotg} \psi d\phi \tag{7}$$

$$q = q_0 + \int_{\phi_0}^{\phi} (q-p) \operatorname{cotg} \psi' d\phi \tag{8}$$

If a principal stress-direction line happens to be straight, i.e., a symmetry axis, then the formula (7) is used in a different form: namely, in the form as given by equation (9):

$$p = p_0 - \Delta\phi \int_0^A \frac{p-q}{\Delta y} ds_1. \tag{9}$$

The same formula is preferably adopted in all cases where the angle ψ, equation (7), is small, because a slight error in ψ makes a large error in cotg ψ.

From this it is seen that theoretically an optical set consisting of two polarizing prisms, including necessary lenses etc., is all that is required for a complete analysis. However, even the simplified procedures as suggested by Coker and Filon are rather complicated. They require for an accurate separation of the p and q stresses, that the p-q values be given accurately for a larger number of points than is ordinarily necessary for practical purposes and furthermore the graphical integration also involves considerable time. For these reasons, additional measurements have been suggested, for instance, the measurement of R_1 and R_2 as outlined above, which allows p and q to be obtained separately without resorting to any complicated mathematics or graphical integration respectively.

Strain Measurements: a. In the plane of a flat model.

This method consists of measuring by means of a microscope the deformations of small circles or squares, with sides parallel to the xy axes, drawn upon models made of steel or flexible materials such as caoutchouc.[5]

[5] Test pieces of caoutchouc covered with a network of straight lines were used by Stodola to study the stresses in turbine-discs, see paper entitled "Die Nebenspannungen in rasch umlaufenden Scheibenrädern", Schweizerische Bauzeitung, Band 51, No. 32, p. 1269; August 10, 1907.

A small circle deforms approximately to a small ellipse when the model is loaded. The principle stresses can be computed by using equations (10) and (11):

$$p=(e_p+\nu e_q)E/1-\nu^2 \tag{10}$$

$$q=(e_q+\nu e_p)E/1-\nu^2 \tag{11}$$

in which e_p and e_q are the unit strains in the principal stress directions, i.e. in the directions of the major and minor axis of the ellipse. The magnitude of e_p is obtained by dividing the difference between the lengths of the major axis and the diameter of the circle by the diameter of the circle. The value of e_q is obtained correspondingly.

A square becomes approximately a parallelogram when the model is loaded. This transformation can be thought of as consisting of two steps: namely, (1) a transformation of the square to a rectangle, for which equations (10) and (11) can be used assuming for this step that the normal stresses p_x and q_y be principal stresses p, q and (2) a transformation of the rectangle to a parallelogram. The angular change, β, determining the distortion of the rectangle, is a measure of the shear stress s, since:

$$\beta=\frac{s}{G} \tag{12}$$

From this it is seen that by measuring the unit strains in x, y directions and the angular change, the shear stress s and the normal stresses p_x and q_y can be obtained, from which in turn the angle ϕ and the principal stresses p and q can be computed by using equations (14) and (15), see appendix.

b. Measurements of lateral strain of a flat model.

Lateral measurements of the expansion or contraction allow $p+q$ to be computed, since the simple relation, as given by equation (13) exists:

$$p+q=-\frac{eE}{\nu} \tag{13}$$

Such measurements can be made in various manners, for instance, with a lateral extensometer, such as designed by Coker.[6] This apparatus

[6] Coker and Scoble, The Distribution of Stress Due to a Rivet in a Plate. Transaction of the Inst. of Naval Architects, 1913, p. 207 to 218. An improved form of lateral extensometer was used by Coker, Chakko and Ahmed in their investigation of Contact Pressures and Stresses, Excerpt Minutes of Proceedings of the Meetings of the Institution of Mechanical Engineers, London, 1921.

utilizes two needles and a combined mechanical and optical multiplying system.

Curves of $p+q=$constant, i.e., contour lines, can be obtained by employing the interference method, which consists of covering the model with a polished glass plate and of observing interference fringes.

The change of the thickness of a model can also be determined by an interferometer or electrically by an arrangement with two condenser plates, of which one is stationary and the other is in connection with the surface of the model. A change in thickness of the model results in a change of the electrostatic field between the plates which in turn can be measured and calibrated in terms of distances or $(p+q)$ values.

Analogy Methods: The *membrane analogy* enables the experimental study of the distribution of shear over irregular cross-sections of bars subjected to twist. Prandtl,[7] who is the originator of this method, used a soap film instead of a membrane and measured the slope of the surface with respect to the plane of the edges by an optical method. This slope is proportional to the shearing stress acting at the corresponding point of the twisted bar.

The *sand pile*, *hydrodynamical* and *electrical* analogies are other helpful means of solving problems of the same nature.

Lueder's Line Method: The principle of this method is to subject polished models of ductile material to increasing loads until the ultimate shear stress, s_{ult}, becomes so large, that the material fails in shear due to overcoming of the slippage resistance. This causes a deformation in the plane surface. Light, reflected by the model, brings out such surfaces as shadows in the high lights of the surrounding areas.

At the beginning of the failure a short line,[8] or narrow band, starting at a point of concentrated stress is usually first detected; then, in the course of the experiment, the failure of complete areas is observed. The loads from overloaded elements are thrown upon neighboring elements, which also fail and in turn throw the load upon the next element. In this manner a continuous redistribution of the stresses takes place. For this reason, it cannot be expected that the contours of the areas in which failure occurred will represent accurately lines of equal ultimate shear; the subsequent stages of failure give an

[7] Prandtl, Phys. Zeitschrift 1903, p. 758. Anthes, Dinglers Polyt. Journal 1906; pg. 342

[8] Such lines were first detected by Lueder. see Dinglers Polyt. Journal 1854.

approximate picture of the distribution of the ultimate shear stress only.

In cases where the stress distribution depends upon the elastic properties of the material and where the results shall be applied to steel structures the method has an advantage in that the tests are made with steel models. The method also lends itself to a partial solution of three dimensional stress distributions, for which the photoelastic method, for instance, probably will fail to give correct results.

As the naked eye is not sensitive enough, particularly in observing the very beginning of the failure, the experimenter is in need of optical equipment, which enables him to detect this beginning and which records subsequent stages. Special optical means have been employed by Nádai[9] to secure satisfactory photographs of the test piece after testing.

General Remarks and Conclusions: Most of the experiments involved in the methods mentioned above are not of a simple nature. It has, therefore, been customary in the past to combine the results of the simplest experiments, such as the $p-q$ values from photoelastic tests with the $p+q$ values from lateral strain measurements, which combination allows p and q to be separated in a simple manner. In making the $p+q$ measurements an interferometer is employed extensively by Mesnager,[10] whereas Coker[6] devised, specially for this purpose, the lateral extensometer mentioned before. The method of solving the problem exclusively by the photoelastic method without resorting to a graphical integration or strain measurements is a very recent development in the photoelastic laboratory of the Technical University in Zurich.[11]

From the above it is obvious that the methods of experimental elasticity make extensive use of optics and it is suggested that this interesting field be given some attention by the optical industry.

APPENDIX

In this appendix a few fundamental considerations of elasticity are outlined for the purpose of making the contents of the paper more intelligible and complete.

[9] Nádai, Der bildsame Zustand der Werkstoffe, Springer 1927, p. 62.

[10] Mesnager, Détermination Complète sur un Modèle reduit des Tensions qui se produiront dans un ouvrage; Annales des Ponts et Chaussées, 1913.

[11] Favre, Méthode purement optique, Schweizerische Bauzeitung, Band 90, No. 23 and 24, 1927.

Fig. 1 shows an orthographic view of a structural element defined by its contour and two parallel surface planes. A small element is cut out with two sides parallel, and four sides perpendicular to these two

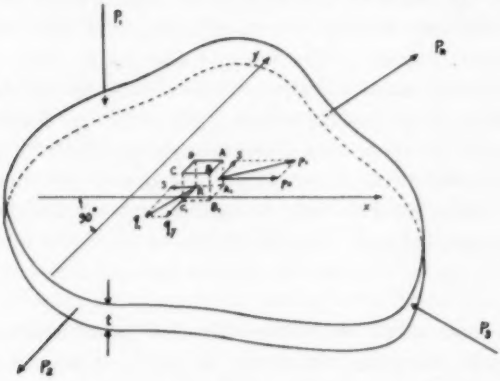


FIG. 1. *Small element showing application of stresses.*

surface planes and with its axes parallel to the x, y axes. The stresses p_1 and q_1 acting parallel to the two surface planes on the four sides AA_1 BB_1 and BB_1 CC_1 etc., will, in the general case, not be perpendicu-

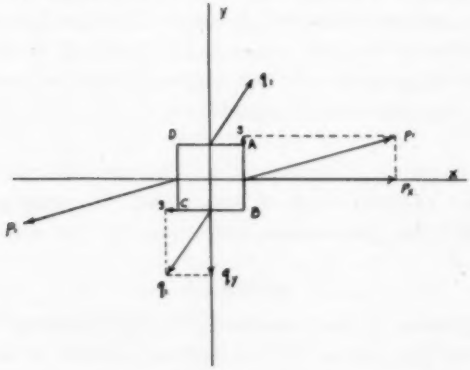


FIG. 2. *Top view of small element Fig. 1.*

lar to these sides, but will be inclined. They are usually resolved into two components: namely, into the normal components p_x and q_y and the shear component s. In Fig. 2, for simplification, only the top view is

shown of the same element as in Fig. 1 in the same position. There exist, however, two particular angular positions of the element which are of special importance. One of these positions, Fig. 3, is characterized by

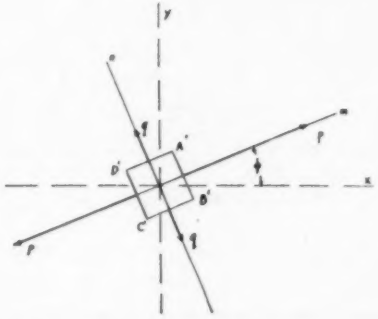


FIG. 3. *Element oriented so that the normal stresses are principal stresses.*

the fact that the normal component p_x has reached a maximum value p, after rotating through the angle ϕ, given by equation (14):

$$\phi = \frac{1}{2} \arctan \frac{2s}{p_x - q_y} \tag{14}$$

The other normal component q_y has become a minimum q and the shear component has vanished. The stresses p and q are called *principal stresses;* their magnitude is given by equation (15):

$$p, q = \frac{p_x + q_y}{2} \pm \frac{1}{2} \sqrt{4s^2 + (p_x - q_y)^2}. \tag{15}$$

The axes m, n of the element in this particular angular position are called the *principal stress directions.*

The integration along straight lines parallel to the x and y axes requires the shear stress s acting on the elements in x, y directions to be known. Its magnitude is given by the equation:

$$s = \frac{p - q}{2} \sin 2\phi \tag{16}$$

From this equation it will be seen that s depends only on quantities which can be evaluated by the photoelastic method with the set ordinarily used.

The other position of the element of special interest is with its axes inclined 45 degrees with respect to the principal stress directions, Fig. 4. For this position all four inclined stresses p_2 and q_2 are of equal

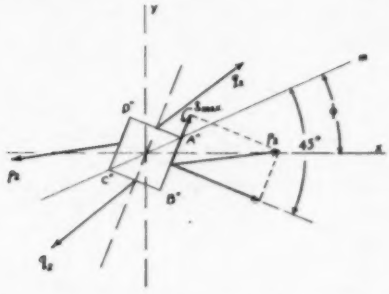
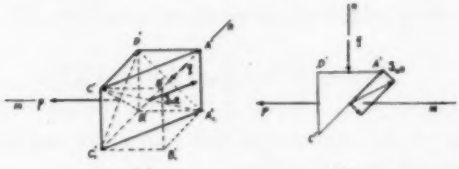


FIG. 4. *Element oriented so that the shear stress s, Figures 1 and 2, is a maximum* s_{max}.

magnitude and the shear stress is a maximum s_{max} depending on the difference $p-q$ between the principal stresses, since:

$$s_{max}=\frac{p-q}{2} \tag{17}$$

In case p and q have different signs, Fig. 5, the maximum shear as given by equation (17) is also the ultimate shear stress of the element.

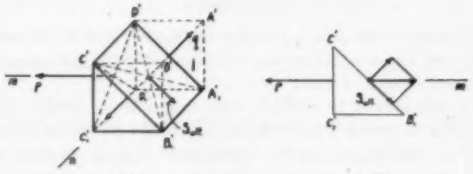


(a) (b)

FIG. 5. *Condition showing* $s_{ult}=(p-q)/2$.

If, however, the signs of p and q are the same, then the ultimate shear stress will be equal to $p/2$, Fig. 6a and 6b or $q/2$, Fig. 7a and 7b, depending upon whether p is larger or smaller than q. From this it is seen that the ultimate shear stress may act either parallel to the two surface planes of the structural element, Fig. 5, or in a plane through the direction of the smaller principal stress and inclined 45° to the surface planes of the structural element, Fig. 6 and 7.

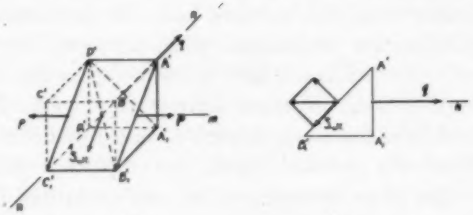
It has been mentioned in the paper that for ductile material the beginning of failure is due to overcoming of the slippage resistance by the ultimate shear stress exceeding a certain value. It is, therefore,



(a) (b)

FIG. 6. *Conditions showing* $s_{ult}=p/2$.

obvious that p and q must be known, since the ultimate shear stress will be equal either to $(p-q)/2$ or $p/2$ or $q/2$ as mentioned above. In brittle material the fracture occurs due to overcoming of the cohesion



(a) (b)

FIG. 7. *Conditions showing* $s_{ult}=q/2$.

resistance and it is evident that in this case also the stresses p and q at all points should be known, especially for those points where concentrated stress exists.

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FURTHER DEVELOPMENT IN PHOTOELASTICITY

By R. V. Baud

SYNOPSIS

It has been customary in the past to machine the models used in photoelastic work either of glass or celluloid and to use a compensator for measuring the difference $p-q$ between the principal stresses p and q, point by point.

In this paper a new method is outlined, consisting of using models made of a material of a large index of forced double refraction and of making photographs or drawings of the isochromatics of the loaded model and of a comparison strip in bending. By matching the colors of the model with those of the strip in bending, a complete quantitative record of the shear stress distribution throughout the model is obtained.

An example of the application of this new procedure is given for the case of a plate with a hole subjected to tension along one axis. Since the theory of elasticity offers an accurate solution for this particular case it can be used as a means of confirmation. The paper shows a very good agreement between analysis and experiment.

Introduction. The photoelastic method of studying stress distributions in transparent materials is based upon the phenomenon that isotropic materials become aeolotropic when subjected to a system of external forces. Since ordinary light is too complex for an analysis of forced double refraction, polarized light is employed. Plane or circularly polarized light traveling through a loaded model is transformed hereby into elliptically polarized light. An elliptically polarized light-ray can be thought of as consisting of two rays polarized in two planes perpendicular to each other; these planes are defined in our case by the direction of the light propagation and the directions of the principle stresses p and q of the element in consideration. The velocities of the two rays depend upon the state of stress of the element. A ray vibrating in the direction of a tensile stress has a larger velocity than a ray vibrating in an unstressed element.[1] On the other hand, the velocity of a ray vibrating in the direction of a compression stress is smaller than the velocity of a ray vibrating in an unstressed element.[1] Since p and q are not equal as a rule but vary throughout the model, it is evident that the two rays travel through its elements with different velocities and that therefore a certain time phase separation R_m between the two rays is produced for each element, which depends upon the material of the model: optical constant c,[2] the thickness t of the model, and the stress-difference $p-q$, in the simple manner as expressed by equation (1).

[1] There exist amorphous solids which behave opposite to this.

[2] Constant c is the difference n_e-n_0 between the indexes of refraction n_0 and n_e of the ordinary and extraordinary ray per unit stress; it is called "Stress-optic coefficient" or "Spe-

$$R_m = c(p-q)t \qquad (1)$$

This equation allows the evaluation of $p-q$. Since the magnitude of the maximum shear stress s_{max}, acting in a plane perpendicular to the model, is equal to $(p-q)/2$, and since shear failures may occur in such planes in case of excessive values of s_{max}, the information obtained regarding the magnitude of $p-q$ in various elements of the model is of great practical importance.

In this paper three photoelastic procedures to evaluate $p-q$ values on basis of equation (1) are described. The available methods to affect a separation between p and q, when $p-q$ is given, and other experimental methods of obtaining the principal stresses p and q are outlined in another paper.[3] In this paper it is explained for what reasons the stresses p and q should also be known separately.

Exact Method: In special cases, where accuracy of experiment is the chief consideration, a monochromatic light source, plane polarized light, and a model machined of glass are used as a rule. The model is located between two crossed polarizing prisms preferably in such a manner that the planes of polarization make angles of 45° with the principal stress directions of the element in consideration. This position results in maximum intensity of the light passing through the analyzer. A crystal-compensator is then placed between the model and one of the prisms so that the axes of the compensator coincide with the principal stress directions. By making the time phase retardation in the compensator R_c equal to R_m in magnitude but of opposite sign and by measuring R_c, the difference $p-q=R_m/ct$ is obtained, whereby the value of the constant c has to be established by an additional experiment, in which p and q are given quantities.

Since making R_m equal to R_c, but of opposite sign, has the same optical effect as having the model alone with the element unstressed, and since an unstressed element of a model between crossed polarizing prisms results in a black spot on the screen, then $R_c=R_m$, if the element shows black on the screen. This procedure is called the compensation method. By compensating is therefore meant optically to cancel the effect of the model by introducing an additional similar effect, but of opposite sign, so that no light passes through the analyzer.

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cific double refraction" or "Index of temporary double stress refraction." In this paper the expression "Index of forced double refraction" has been adopted.

[3] Baud, Experimental Methods of Studying Stress Distributions, see page 411 of this issue of the Journal.

Commercial Method: In commercial work the ease of the material with regard to machining becomes of importance on account of the complicated shapes to be tested. Camphor-celluloid is very favorable in this respect and has therefore been used extensively since 1910. It is commercially available in sheets of suitable thickness.

The test procedure is simplified by using circularly polarized light, since in this case the directional bands are eliminated and the angular position of the model is independent with regard to the axes of polarizer and analyzer. This requires, however, a monochromatic light source, which emits light of just the proper wave length. Since a powerful light source of this type is not available as a rule, an incandescent lamp is usually used in commercial photoelastic work, although the light becomes elliptically polarized, since an incandescent lamp emits a continuous spectrum. For elliptically polarized light, the colored image of the model on the screen is not truly representative.[4] It is generally assumed, however, that with the compensation method the error thus introduced is compensated.

To obtain quick results a strip machined from the same sheet as the model axially loaded in tension, p_c, is used as a compensator. The angular orientation of the strip is so that its axis coincides with one of the principal stress directions of the element tested. If a lag R_m is present in the model in one principal direction, a lag R_c of the same amount is produced in the comparison strip in the direction at right angle. Fig. 1_a to 6_a show various states of stress of an element A of the model and Fig. 1_b to 6_b show the corresponding direction and magnitude of the tensile stress p_c of an element B in the compensation strip.[5] This

[4] A filter is sometimes added to the optical set to eliminate undesirable wave lengths. Monochromatic light has a disadvantage, however, in that a change of the intensity instead of a change in the color is obtained. Furthermore, by employing such light, black bands are observed which connect elements having $p-q=0$, λ/ct, $2\lambda/ct$, $n\lambda/ct$, whereas when white light is employed the elements having $p-q=0$ are the only elements for which a black spot is obtained.

[5] An error is theoretically introduced in that the thickness, which is a function of the state of stress in each element, in general will be different in model and compensation strip. Assuming an element of the celluloid model in compression, so that $p-q=1500$ lbs. per sq. in., we obtain with $E=300{,}000$, $\nu=0.4$, $t=.200''$ an expansion of $0.0004''$ and a contraction of the compensation strip of the same amount. The total change in thickness is less than one thousandth of one inch. The error introduced is, therefore, insignificant, inasmuch as there may be initial variations of the thickness of the same order, which are also neglected in commercial photoelastic work.

The strip is subjected to tension, instead of compression, for the reason that it can be produced more uniformly and by less complicated means.

method is much simpler than the accurate method for the reason that the knowledge of the constant c is not required, since $cp_c t = c(p-q)t$, i.e. $p_c = p-q$; p_c is obtained by dividing the load on the strip by its cross-sectional area.

The two elements A and B can also be thought of as being one element only with the stresses superimposed. By compensating is then meant

FIG. 1.

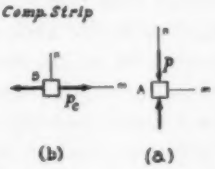

FIG. 2.

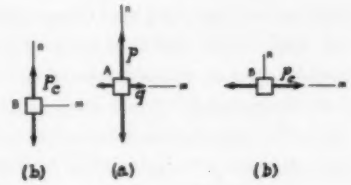

FIG. 3.

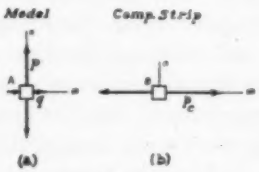

FIG. 4.

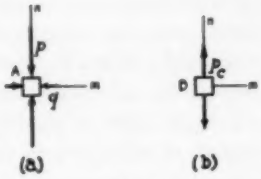

FIG. 5.

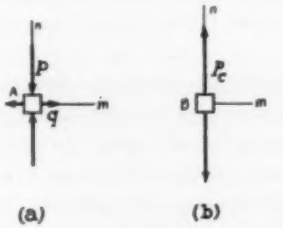

FIG. 6.

FIG. 1–6. *Method of compensation by means of a celluloid strip in tension.*

elastically, making p_c so that the difference between the resultant superimposed stresses in the two directions is equal to zero.

For a given loading of the model, one compensation must be made for each point. Since camphor-celluloid has only a moderate index of forced double refraction, the picture of the model at zero load is usually not reliable enough to conclude that there is no initial double refraction and it therefore becomes necessary to make the compensation at least twice, with full and zero load. Since the compensation of small amounts

of double refraction is very inaccurate, a method has been adopted by us, consisting of making several compensations at medium and large loads and of plotting the readings of p_e against the values of the loads, which results in a straight line. The true value of p_e, or $p-q$ respectively, for any load is equal to this load multiplied by the tangent of the angle of inclination of the straight line. This procedure also avoids possible inaccuracies in the zero settings of the scales used to measure the loads on model and compensation strip. It requires, however, that an individual series of measurements be made for each point. If a mathematical separation of p and q stresses be contemplated, the knowledge of $p-q$ in a very large number of points is required in order that the separation be successful. There exists, however, a tendency and necessity to reduce the number of points as much as possible, since such measurements are a severe strain on the eyes and require considerable time, and since camphor-celluloid exhibits some undesirable time-effects, such as evaporation of camphor along the edges. The number of points, where $p-q$ are taken, will be limited for these reasons and therefore for all other points no record of their state of stress is obtained. Sometimes in working up experimental data for actual use in the design of machines or comparing them with analytical results, it so happens that the magnitude of $p-q$ should be known for certain points, for which no readings have been made and on the other hand for other points data are obtained, which may subsequently prove to be unimportant.

The method of projecting properly the model as well as the compensation strip upon the screen for superposition of the pictures requires two parallel beams of polarized light, obtained by means of a comparatively very elaborate system of expensive lenses. It is obvious that a method which requires only one parallel beam of polarized light is a decided advantage, especially for all cases where allowances for such work are restricted. Such a method will now be described.

New Commercial Method: The new features consist of (a) using a material having a large index of forced double refraction, such as bakelite, (b) comparing colors instead of compensating colors, (c) making in various manners records of the isochromatics of the model and of similar isochromatics of a comparison strip of the same material, subjected to a known bending moment. Bending of a strip offers decided advantages over straight tension since zero stress is well defined by the neutral axis and a wide range of stresses, from a maxi-

mum tension to a maximum compression is obtained at one time. Where the sign of the stress changes, a dark line surrounded by a narrow band of dark grey with an olive shade is observed.[6] With increasing stresses the colors appear in the following succession: yellowish olive, yellow, orange, red, brown and green which repeat several times with the exception of dark gray and yellowish olive. Beyond the fourth order the colors become less distinctive.

It has been noted that within the proportional limit of commercial camphor-celluloid of 1/4″ thickness, only colors of the first order, and in some cases yellow of the second order can be obtained, whereas between six to eight color orders are observed within the proportional limit of bakelite of the same thickness. From this it is seen that by increasing the stress in bakelite the colors repeat more frequently and therefore only a small stress-range is represented by the color of a certain order. For instance, yellow of the first order represents roughly a stress-range of only 30 to 165 lbs./sq. in. as compared with 200 to 700 lbs/sq. in. in camphor-celluloid. For other colors of the corresponding order a proportional difference in the range is observed. From this it can be concluded that an error in judging the color-shade is small in case bakelite or any other material with a large index of forced double refraction is used, whereas a similar error is large in terms of stress in the case of commercial camphor-celluloid, which has only a moderate index of forced double refraction.

There are three methods to get a permanent record of the stress distribution of the bent strip and the model. The simplest method consists of drawing a few well defined isochromatics by hand upon a sheet of paper, placed so that a suitable enlargement is obtained. A colored photograph can be made instead, in sizes up to 7″ x 10″. The third method consists of making an ordinary black and white photograph. Although such a photograph does not allow as accurate a descrimination in the magnitude of shear stresses as the colored one does, it has the advantage that an ordinary photograph is more convenient for purposes of reproduction.

[6] These remarks are strictly true for the photoelastic apparatus in the Research Laboratory of the Westinghouse Electric & Mfg. Co., which at present is used with an incandescent lamp without color filter. Tests have been made with this apparatus, consisting of rotating a bent bakelite strip and of studying the shift of the isochromatics. These tests indicate that an average error of roughly 4% is introduced, when the color distribution of the strip in vertical position, i.e. in the plane of the polarizer, rather than various angular positions is used as a basis of comparison.

The order of the isochromatics of the model can easily be established by increasing the load from zero to the one desired and counting the number of times a certain color appears. If the order of one of the isochromatics be determined, it is a simple thing to get the order of all others, because the colors and orders appear without discontinuity.

Application of new method to case of tension member with hole: The new method was applied for the first time to obtain lines of constant maximum shear stress for a plate with a hole, in tension, for which case on exact solution is available.

a. Design of model and comparison strip.

The dimensions of the model and comparison strip are given in Figs. 7 and 8. It was first intended to make the ration of the width W

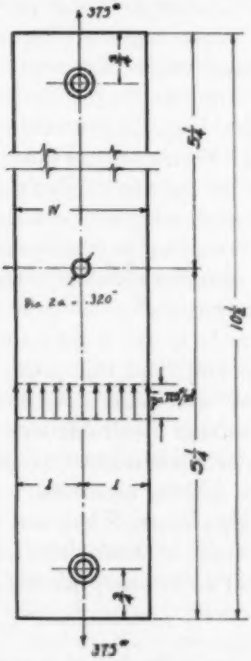

FIG. 7. *Test specimen subjected to tension along main axis.*

of the tension member to the diameter $2a$ of the hole equal to 8. The width W of the model was limited to 2" by the width of the parallel beam of polarized light. The drilling of a hole of 1/4" diameter produced a set of initial stresses. In order to get rid of them the hole had to be

reamed out. The model had finally a ratio $W/2a = 6\text{-}1/4$ only. In certain respects a larger ratio would have been a decided advantage, the reason for which is mentioned later on in the paper.

b. Material. Model and comparison strip were machined from the same sheet of bakelite, having a thickness of .261".

For the central part of the tension member the material was exceptionally good, even without annealing, whereas the material for the comparison strip had a slight initial double refraction, which, however,

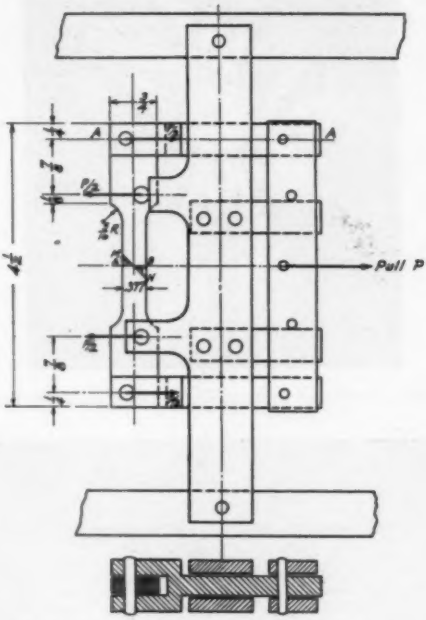

FIG. 8. *Loading arrangement used to produce bending of the comparison strip.*

could be taken care of by making a correction as is described in Footnote 8.

c. Loading. The model was subjected to a tensile load of 375 lbs. producing an average stress equal to $\bar{p} = 720$ lbs. per sq. in., see Fig. 7.

A bending moment of 13.12 in-lbs. was applied to the comparison strip[7] by the loading arrangement shown in Fig. 8. This resulted in a bending stress $p_b = 2140$ lbs/sq. in. in the extreme fibers.

[7] Strip was placed in a vertical position, see Footnote 6.

d. Results. Colored and black and white pictures were made from the loaded model and from the comparison strip in bending, see Figs. 9 and 10.

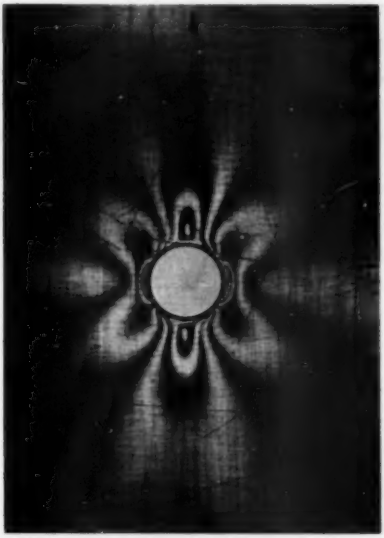

FIG. 9. *Stress distribution in test specimen (black and white photograph).*

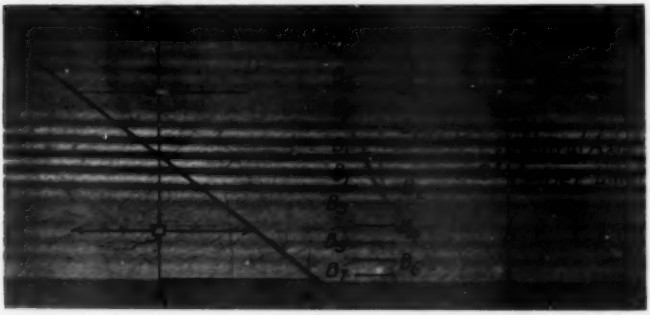

FIG. 10. *Stress distribution in comparison strip (black and white photograph).*

The color-boundaries of the model, i.e. the lines where the colors distinctly change, were traced from the colored plate for all four quadrants. The average distances from the center were taken and the isochromatics for one quadrant as shown in Fig. 11 were obtained.

Similar color-boundaries were selected on the colored photograph of the comparison strip, their distances being measured from suitable reference lines. Fig. 12 shows the cross-section AB of the comparison strip enlarged approximately 9 times. Since the stresses in the extreme fibers,

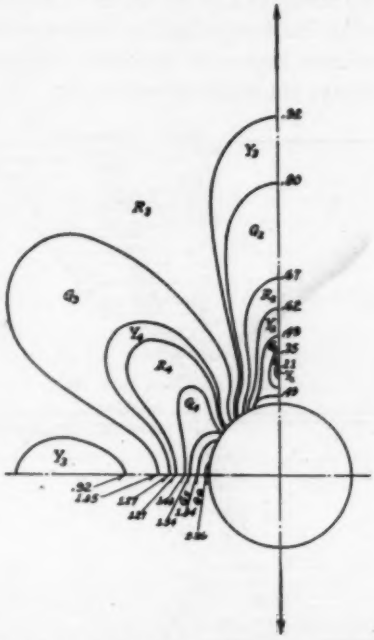

FIG. 11. *Color boundaries from colored plate. (Abbreviations: Yellow—Y, Red—R, Green—G. The sub-letters denote the color order. The figures represent the ratios r.)*

vectors AM and BN, are known from computation[8] and since the stress distribution follows a straight line MN the stress p is given directly for each isochromatic. The p value thus established for each isochromatic

[8] It will be noted from Fig. 10 that the neutral axis does not coincide with the center line of the comparison strip, but has an eccentricity e toward the upper edge, which was in compression. When the bending moment M was increased from zero, it was noted, that the neutral axis gradually shifted from the upper edge toward the center line, indicating that there was an initial double refraction, which acted like an initial tensile stress field, with the stress acting in a longitudinal direction and of a magnitude p_T, which may be assumed to be constant for the cross-section AB. The magnitude of p_T can be computed in this case since $p_T : p_b = e : b$. From this relation we obtained $p_T = 90$ lbs. per sq. in. By including this correction, the stresses at the outer fibers, vectors AM and AN, became $2140 + 90 = 2230$ and $2140 - 90 = 2050$ lbs. per sq. in. respectively.

of the comparison strip is equal to the magnitude of the stress-difference $p-q$ for the isochromatic of corresponding order on the model. For instance, the isochromatic between 3rd order red and green of the comparison strip represents 750 lbs/sq. in. in tension and 760 lbs/sq. in. in compression, or an average of 755 lbs/sq. in. This is equal to the stress difference $p-q$ for the line separating the 3rd order red and green on the model. For convenience, instead of the stress difference itself, the ratio r of $p-q$ to the average stress $\bar{p}$ is given in Fig. 11. This is more con-

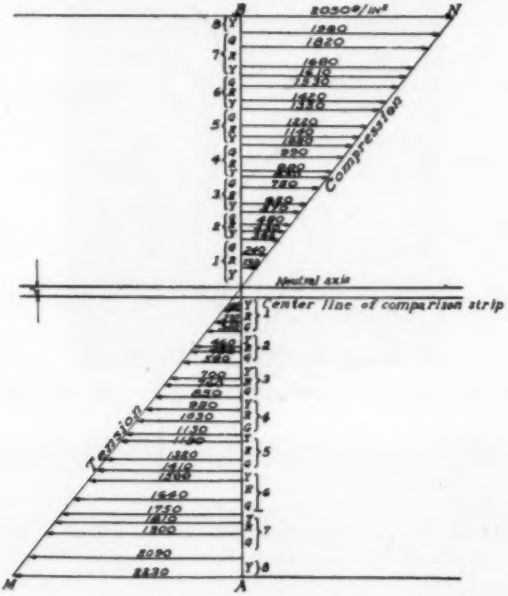

FIG. 12. *Stress distribution in comparison strip from colored plate.*

venient because $p-q$ varies with the load, but the ratio r will remain the same within the elastic stage. In the above case $r=755/720=1.05$. The ratios r for all other isochromatics were obtained in a similar manner.

If instead of colored pictures, black and white ones are made, it is suggested that lines of maximum darkness and brightness be selected and traced. This was done in our case and by taking average distances from the center for the four quadrants as before, the lines shown in Figs. 13 and 15 resulted. In a similar manner the stress distributions

for the comparison strip were obtained. Fig. 14, for instance, shows the stresses as represented by the centers of the black bands.[9]

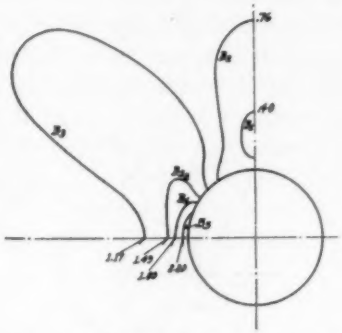

FIG. 13. *Lines of maximum darkness from Fig. 9.*

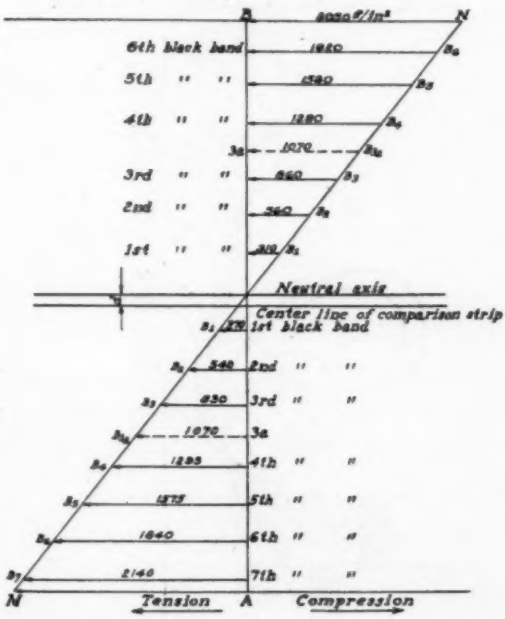

FIG. 14. *Stress distribution in comparison strip from Fig. 10.*

[9] The figure containing the stress distribution with the stresses as represented by the centers of the bright bands is omitted as this figure would not show anything principally different from Fig. 14.

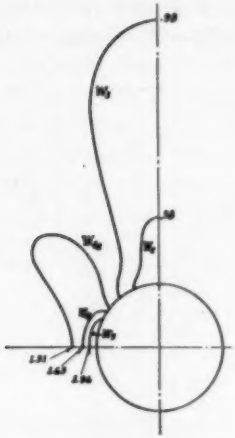

FIG. 15. *Lines of maximum brightness from Fig. 9.*

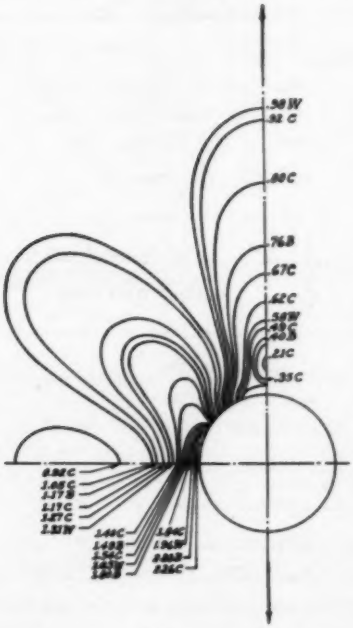

FIG. 16. *Lines of constant ratio r as obtained by new method by superposing Figs. 11, 13, and 15.*

The information derived from colored and black and white photographs can finally be combined. In doing so, the lines and the ratios r as shown on Fig. 16 were obtained. At first, some inconsistencies were noted in that the ratio r did not increase or decrease steadily but erratically. The impression was, that one black line on the picture of the comparison strip had not been observed. In looking into this it was detected that all black lines on Fig. 14 are about equally spaced with the exception of the third and fourth, which show about twice the regular space between them. To further study this the stress distributions over the cross-section AB of the comparison strip were combined. It was detected that dark green, as a rule, produces maximum darkness;

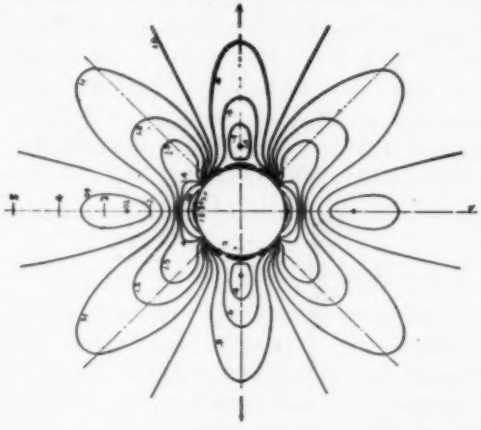

FIG. 17. *Lines of constant ratio r as computed.*

furthermore, that green of the fourth order did not produce black on the black and white picture of the comparison strip but did so on the black and white picture of the model. This explained fully the inconsistencies in the ratios r mentioned and which disappeared by introducing in the stress-distribution of the comparison strip the black line $B3a$, (see Fig. 14) and two white lines $W4a$ and $W4b$.

Comparison of experimental and analytical values of $p-q$. The theory of elasticity gives for the case of a hole of small diameter in a large plate an accurate solution in form of a stress function F[10]. Since in our case the ratio $W/2a$ is only 6-1/4, the stress function F is not necessarily

[10] Foeppl, Drang and Zwang, *1*, p. 315.

a good approximation in this case, although it is generally assumed, that it is accurate for a ratio 8, which is not much larger.

The normal and shear stresses for an element oriented with one of its axes radial can be obtained by derivation of the stress function. With these stresses known it is a simple matter to obtain the difference $p-q$

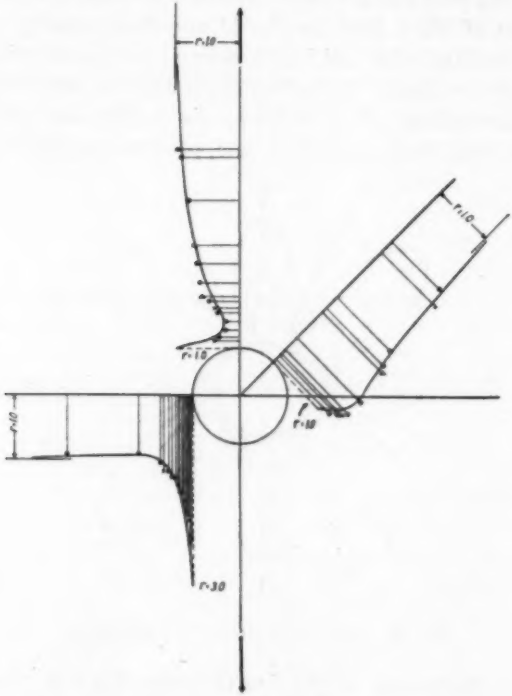

FIG. 18. *Comparison of computed and experimental results:*
— *computed.*
⊙ *Values from color boundaries.*
× *Values from lines of maximum darkness.*
△ *Values from lines of maximum brightness.*

between the principal stresses and also the ratio r as before. Lines of $r=$ constant are shown on Fig. 17[11] and it will be noted that they agree well in form with those obtained from experiment, Fig. 9.

[11] The calculation of the lines $r=$constant, was carried out by Mr. A. M. Wahl of our Research Department.

The magnitudes of the computed ratios r and those experimentally obtained can now be compared for any section. For instance, the full lines on Fig. 18 represent the calculated curves for the two sections through the axes of the model and a section inclined 45°. The points shown represent the values of r as obtained by the new method. It will be noted, that the experimental data are in very close agreement with those computed, using the stress function. Since it is very unlikely that this good agreement is an accidental one, it is concluded that the data obtained in these two entirely different manners must be correct, i.e. (1) that the stress function holds within the experimental accuracy for ratios of the width of the plate to the diameter of the hole as low as 6-1/4 and (2) that the new procedure gives accurate results. To decide just how accurate they are, additional tests have to be made. It is obvious, however, that the accuracy will meet the requirements of commercial work, provided material with little initial double refraction is available. If such material is not available, or annealing does not prove successful, then the initial conditions must be studied first and corrections must be made accordingly. In this case the procedure becomes complicated and no advantages will be gained by the new method.

SUMMARY

In this paper three different photoelastic procedures of determining the distribution of the shear stresses in complicated members are described. The accurate method is used when the accuracy of the results is the chief consideration. For commercial purposes the same method of compensation is generally used as in the accurate method, but in a simplified manner. A new method is explained and applied to obtain the distribution of shear stresses in a tension member with a hole and it is shown that this new method gives satisfactory results. It is based upon the principle of matching colors and has an advantage over the present commercial method in that the work in the dark is reduced largely and the eyes are relieved from undue strain. Furthermore, it does not necessarily require that the principal stress directions be established. Its outstanding features, however, are: Simplicity in equipment and testing, completeness of record, and convenience for purposes of reproduction.

WESTINGHOUSE ELECTRIC AND MANUFACTURING CO.,
EAST PITTSBURGH, PA.

A Source Book in Astronomy. By Harlow Shapley, Director of the Harvard Observatory, and Helen E. Howarth, Research Assistant at the Harvard Observatory. xvi+412 pp. McGraw-Hill Book Co., Inc. New York. $4.00.

The first of a series of "source books in the history of the sciences." This series is being prepared under the general editorship of Gregory D. Walcott, with the assistance of an advisory board of eminent scientists and philosophers, and is officially endorsed by the American Philisophical Association, the History of Science Society, the American Association for the Advancement of Science, and various other societies and associations in special branches of science. The aim of the series is "to present the most significant passages from the works of the most important contributors to the major sciences during the last three or four centuries."

The present book on Astronomy, "although . . . . not to be considered a systematic and connected history of the subject," includes extracts arranged essentially in chronological order from the works of more than sixty contributors to astronomical science from Copernicus to 1900, excluding "the discoveries of living astronomers." So far as possible "the spelling, punctuation and typographical style of the originals have been retained in nearly all instances." The reader has placed before him therefore what is in effect the text of the various contributions upon which the modern science of astronomy has been built. (English translations, of course, are used when the original article was in a foreign language.)

Thus, one finds an extract—some twelve pages—from the "De Revolutionibus Orbium Celestium" of Copernicus, proposing the theory that the earth moves round the sun; Tycho's famous observations on the nova of 1572; Kepler's discovery of the laws of planetary motion; Galileo's account of his discoveries with the telescope; the first observation of a transit of Venus, by Horrox; Roemer's determination of the velocity of light; extracts from Newton's works; Bradley, on the discovery of the aberration of light; William Herschel, on the discovery of Uranus; Adams and Leverrier on the discovery of Neptune; Herbert Spencer, on cosmic evolution; Helmholtz, on the source of the sun's heat; Langley, on the sun's radiation; Lockyear, on stellar evolution—to mention only a few of the more important articles.

"It is surprising how innocent the present-day astronomer is of first-hand knowledge of the classics of his science" say the authors in the preface. The statement applies with equal generality if "astronomer" be replaced by "scientist." Indeed, the reviewer would go much farther and say that the average *educated* public knows far more about the Napoleonic wars, than about the evolution of our concepts concerning the solar

and the stellar systems. Perhaps this condition is only natural, for information about the Napoleonic wars is easily accessible—is, in fact, regularly taught in courses in history. Copernicus, however, is given three lines in a popular Outline of History!

Books such as the Source Book in Astronomy, however, should do much to remedy this situation. For, the reader—be he professional astronomer, student of any of the sciences, or general layman—will find in the present volume a wealth of fascinating material which, whether it be casually read or carefully studied, requires little piecing together to give a clear picture of the origin of astronomical science.

F. K. RICHTMYER

The Collected Works of J. Willard Gibbs. Two volumes: Volume I, Thermodynamics, xxviii+434 pages; Volume II, Statistical Mechanics; Vector Analysis and miscellaneous subjects, xxix+491 pages. Longmans, Green and Co., New York. Both volumes, complete, $6.00; either volume, separately, $3.75. Postage ten cents per volume.

This is the first complete edition of Gibbs' works. The former edition, published in 1906, three years after the author's death, under the title "The Scientific Papers of J. Willard Gibbs," did not include the important treatise on "Elementary Principles in Statistical Mechanics," originally published in book form. This treatise is included as Part I of Volume II of the present edition, the remainder of which is a photographic reproduction of the 1906 edition.

Following a brief biographical sketch of the author, by the late Professor H. A. Bumstead, there is included in Volume I, Gibbs' contributions to Thermodynamics—some eight papers published between 1873 and 1897. Among these is the paper "On the Equilibrium of Heterogeneous Substances," published in 1878, which, as Professor Bumstead says "is generally, and probably rightly, considered his (Gibbs') most important contribution to physical science and is unquestionably among the greatest and most enduring monuments of the wonderful scientific activity of the nineteenth century."

As mentioned above, Part I of Volume II is a reprint of Gibbs' book, "Elementary Principles in Statistical Mechanics, developed with especial reference to the Rational Foundation of Thermodynamics." Part II contains some twenty papers under the respective headings: Dynamics, Vector Analysis and Multiple Algebra, The Electromagnetic Theory of Light, and Miscellaneous Papers. Among these papers may be mentioned the following: "On the Fundamental Formulae of Dynamics" (1879); "Elements of Vector Analysis, Arranged for the Use of Students in Physics" (1881) and 1884); "On Multiple Algebra" (1886); "On the Determination

of Elliptic Orbits from Three Complete Observations" (1889); and several papers on double refraction and dispersion.

It would be as presumptuous as it is unnecessary to present a critical review of the writings of J. Willard Gibbs. His name is known far beyond the confines of the science to which he made such fundamental contributions during the latter half of the nineteenth century. Twenty years after its appearance Le Chatelier translated into French the paper "On the Equilibrium of Heterogeneous Substances," giving as the reason for the translation "not the historical interest in the memoir, but the many important questions which it discusses and which have not even yet been worked out experimentally." Much the same remark can probably be made with respect to the present edition. In spite of the fact that they were written decades ago, during the reign of what is now called "classical physics," "there is no doubt that interest in the works of Willard Gibbs and appreciation of their fundamental importance have greatly increased in the twenty-two years since the publication of the former edition." As proof of the fact that the present undertaking is not merely a "labor of love" of historical portent only, but rather is a substantial contribution to *modern* science by thus making the material more widely accessible, may be mentioned the proposed project to publish in the near future a commentary, prepared by recognized authorities in the several fields, to aid the student of Gibbs' writings.

Students of Physics, Chemistry, Mathematics and other sciences, pure and applied, will be grateful to those who have sponsored the preparation of this new edition, particularly to Professor Irving Fisher, of Yale University, through whose generosity "the two volumes are offered at a low price" so as "to make the original text . . . . accessible to a far wider circle of readers than heretofore."

F. K. RICHTMYER